Universitas Pembangunan Jaya

Integrity, Professionalism, & Entrepreneurship

Mata Kuliah : Analisis Struktur
Kode : CIV – 209
SKS : 4 SKS

Deformasi Elastis Struktur Balok dan Portal

(Moment Area Method)

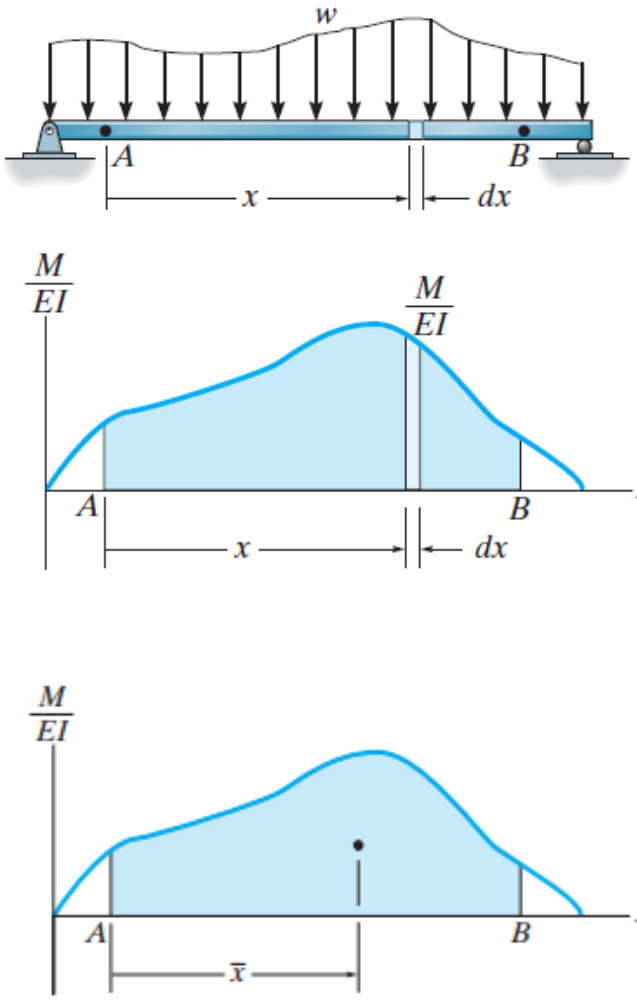
Pertemuan - 4

- Kemampuan Akhir yang Diharapkan
  - Mahasiswa dapat menganalisis deformasi struktur balok dengan metode Luas Momen

- Sub Pokok Bahasan :
  - Metode Luas Momen

Moment Area Theorem

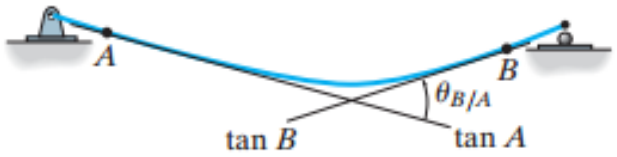
- Metode ini dikenal juga dengan metode Mohr dan cukup unggul penggunaanya untuk mengetahui putaran sudut dan perpindahan vertikal pada titik tertentu suatu struktur akibat rangkaian pembebanan.
- Perhatikan struktur balok seperti terlihat pada gambar 4.1. Akibat pembebanan yang terjadi, maka akan terbentuk kurva diagram momen



Teori I : **The change in slope between any two points on the elastic curve equals the area of the *M/EI* diagram between these two points.**

$$\theta_{B/A} = \int_A^B \frac{M}{EI}\,dx$$

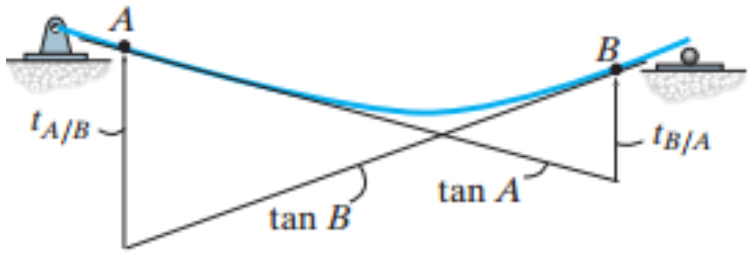
$\theta_{B/A}$ dalam radian



elastic curve
(c)

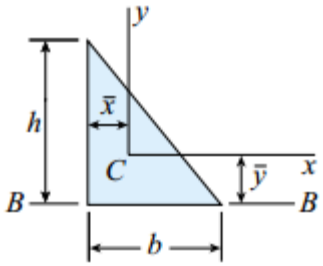
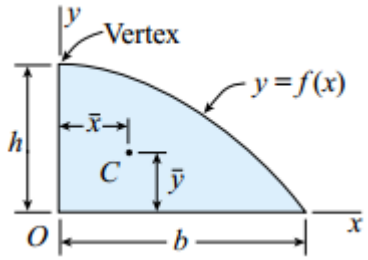
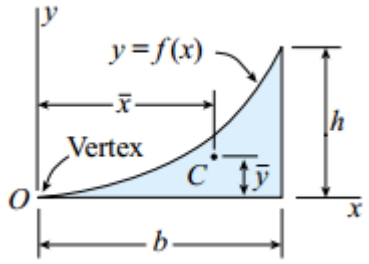
Teori II : **The vertical deviation of the tangent at a point (*A*) on the elastic curve with respect to the tangent extended from another point (*B*) equals the "moment" of the area under the *M/EI* diagram between the two points (*A* and *B*). This moment is computed about point *A* (the point on the elastic curve), where the deviation is to be determined.**

$$t_{A/B} = \bar{x}\int_{A}^{B} \frac{M}{EI}\,dx$$



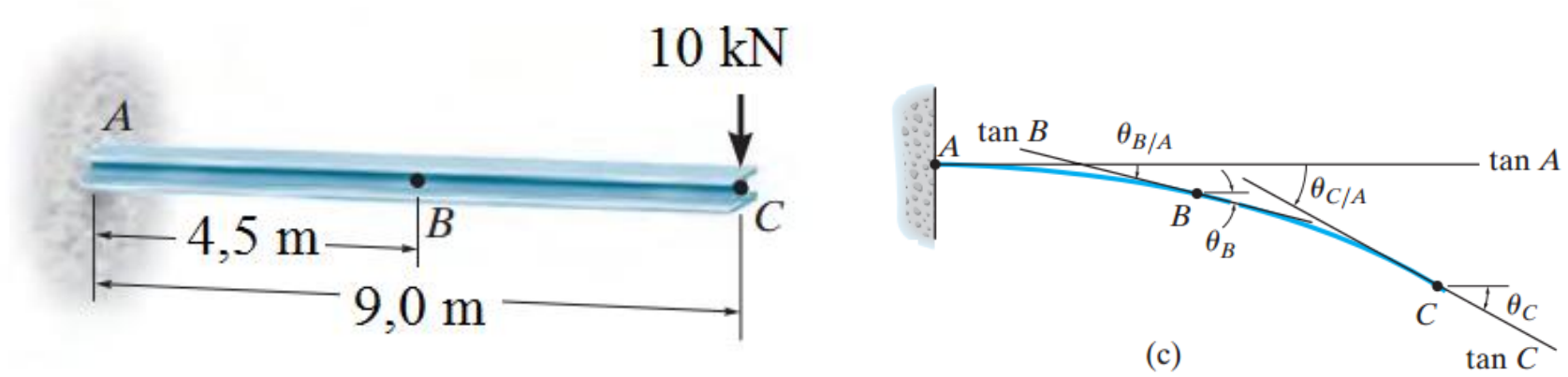
elastic curve

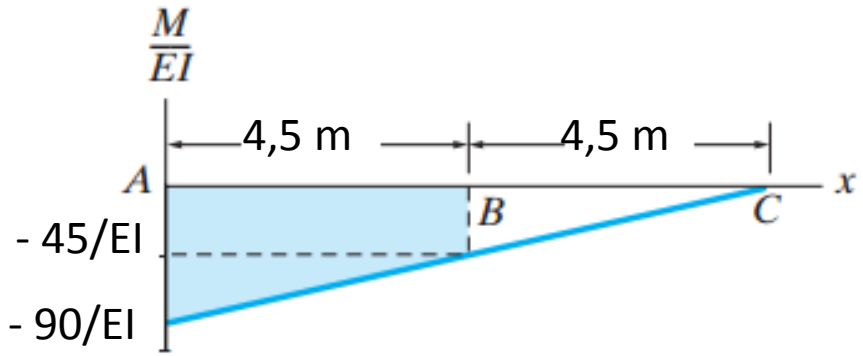
- Note that in general $t_{A/B}$ **is not equal** to $t_{B/A}$
- The moment of area under the M/EI diagram between A and B is computed about point A to determine $t_{A/B}$, and it is computed about point B to determine $t_{B/A}$

SHAPE	AREA	CENTROID
	$A = \frac{bh}{2}$	$\bar{x} = \frac{b}{3}$ $\quad \bar{y} = \frac{h}{3}$
	$A = \frac{2bh}{3}$	$\bar{x} = \frac{3b}{8}$ $\quad \bar{y} = \frac{2h}{5}$
	$A = \frac{bh}{3}$	$\bar{x} = \frac{3b}{4}$ $\quad \bar{y} = \frac{3h}{10}$

Example 1

- Determine the slope at points B and C
- Take E = 200 GPa, I = 250(10^6) mm^4.



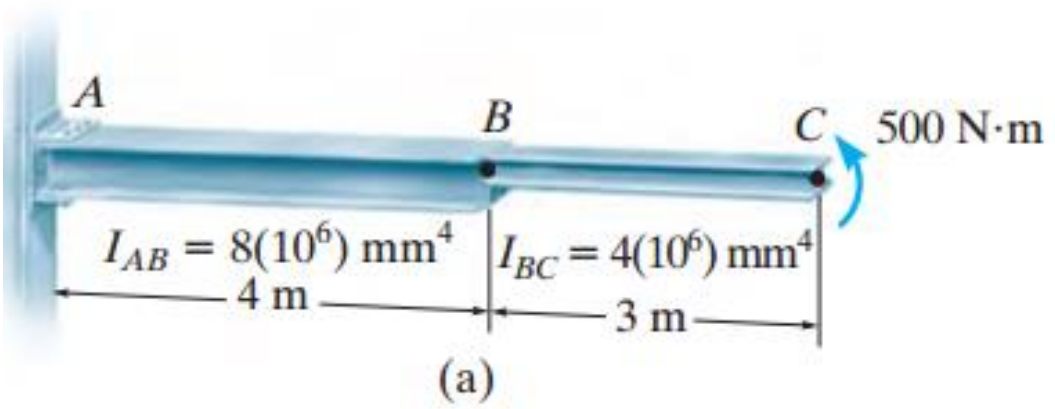


$$\theta_B = \theta_{B/A} = \left(-\frac{45}{EI}\times 4{,}5\right) + \frac{1}{2}\left(-\frac{45}{EI}\times 4{,}5\right) = -\frac{303{,}75}{EI}\,\text{kN.m}^2$$

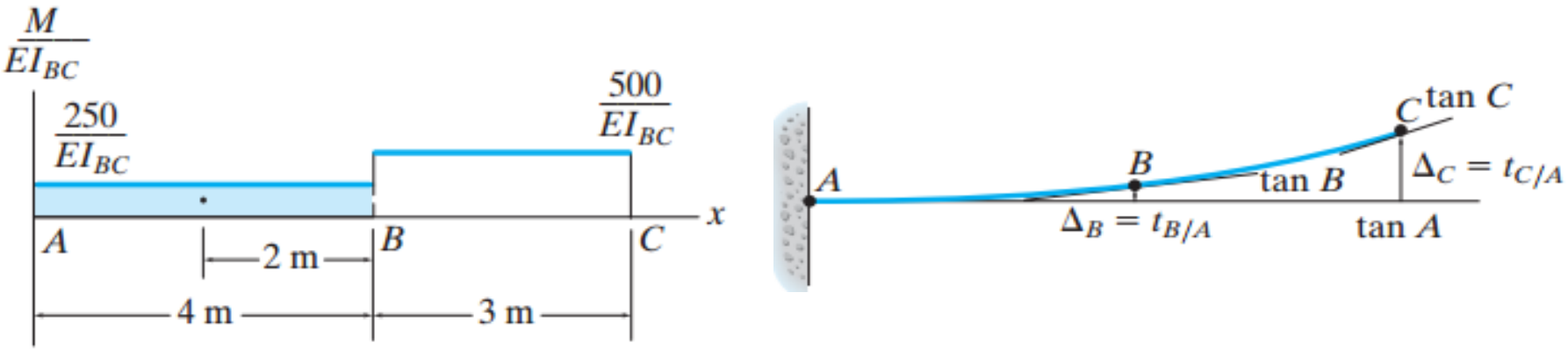
$$\theta_C = \theta_{C/A} = \frac{1}{2}\left(-\frac{90}{EI}\times 9\right) = -\frac{405}{EI}\,\text{kN.m}^2$$

Example 2

- Determine the deflection at points B and C
- Take E = 200 GPa



By inspection, the moment diagram for the beam is a rectangle.
Here we will construct the *M/EI* diagram relative to I_{BC}, realizing that $I_{AB} = 2I_{BC}$

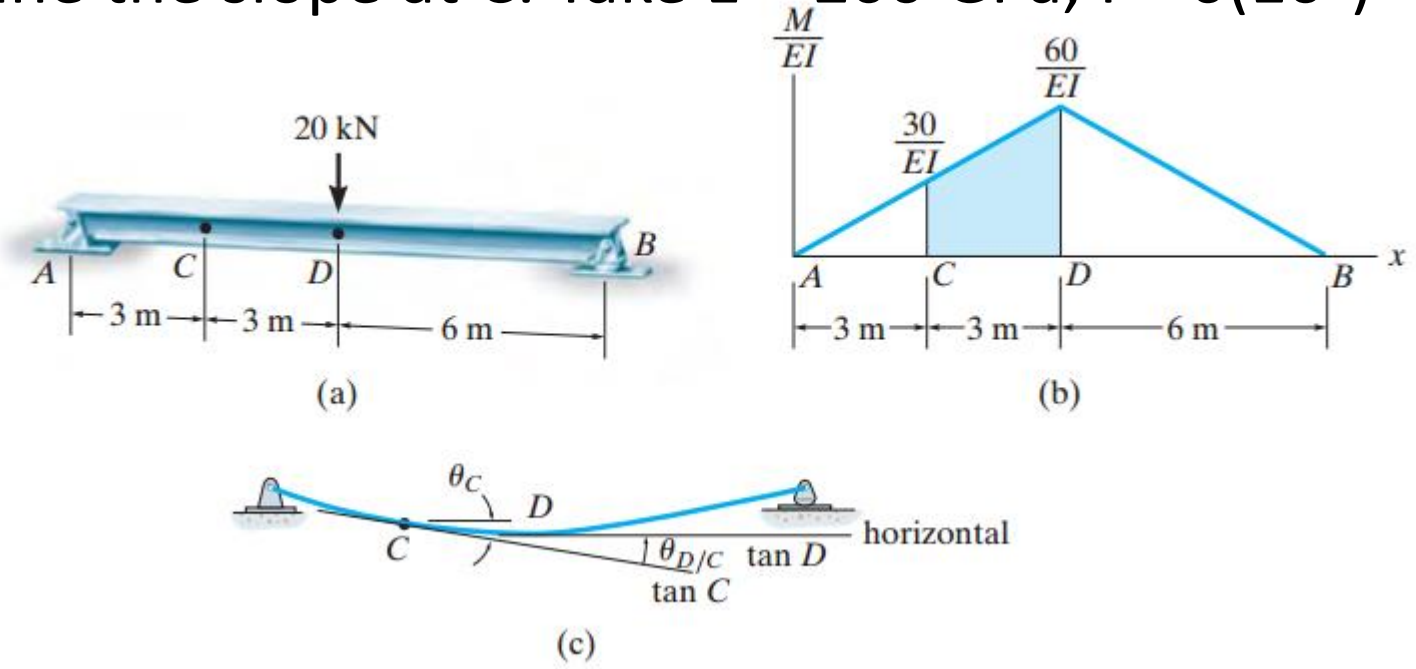


$$\Delta_B = t_{B/A} = \left[\frac{250}{EI_{BC}} \times 4\right](2) = \frac{2000}{EI_{BC}}\ \text{N.m}^3$$

$$\Delta_C = t_{C/A} = \left[\frac{250}{EI_{BC}} \times 4\right](5) + \left[\frac{500}{EI_{BC}} \times 3\right](1{,}5) = \frac{7250}{EI_{BC}}\ \text{N.m}^3$$

Example 3

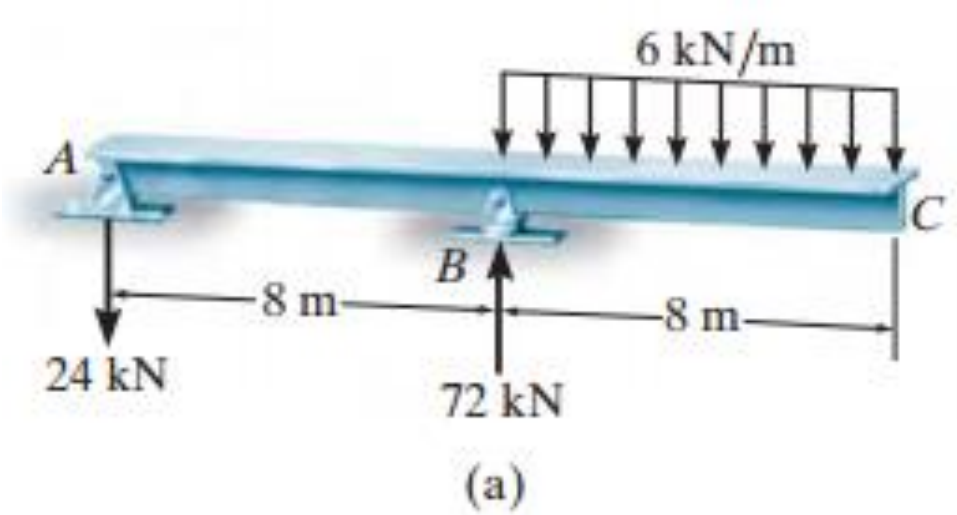
Determine the slope at C. Take E = 200 GPa, I = $6(10^6)$ mm^4.



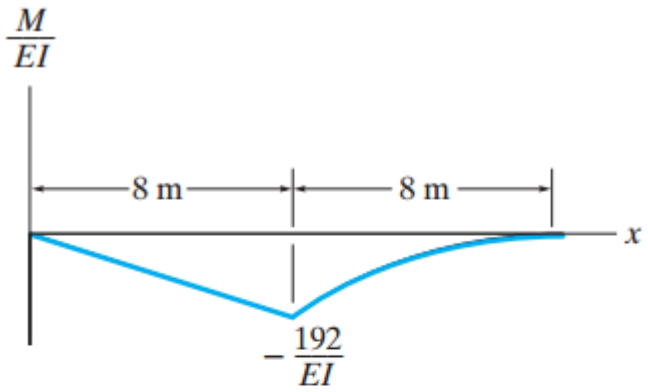
$$\theta_C = \theta_{D/C} = 3\left(\frac{30}{EI}\right) + \frac{1}{2}(3)\left(\frac{60}{EI} - \frac{30}{EI}\right) = \frac{135}{EI}\,kN.m^2 = \frac{135}{200\times 6} = 0{,}112\ \text{rad}$$

Example 4

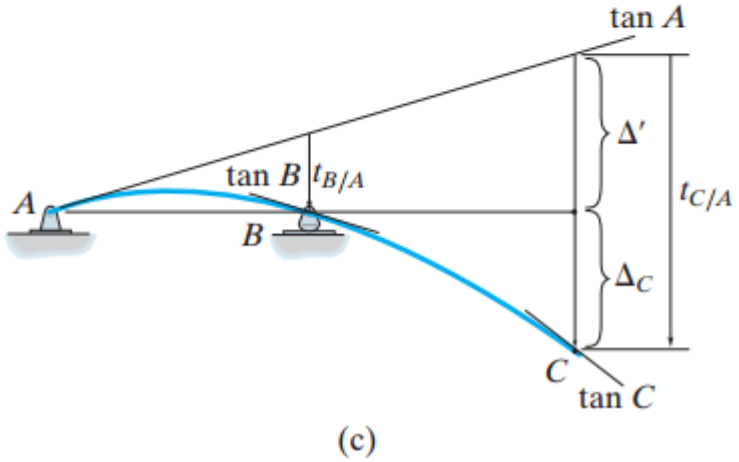
- Determine the deflection at point C
- Take E = 200 GPa, I = 250(10^6) mm^4.



(a)



(b)



(c)

$$\Delta_C = t_{C/A} - 2t_{B/A}$$

$$t_{C/A} = \left[\frac{3}{4}(8)\right]\left[\frac{1}{3}(8)\left(-\frac{192}{EI}\right)\right] + \left[\frac{1}{3}(8)+8\right]\left[\frac{1}{2}(8)\left(-\frac{192}{EI}\right)\right] = -\frac{11.264}{EI}$$

$$t_{B/A} = \left[\frac{1}{3}(8)\right]\left[\frac{1}{2}(8)\left(-\frac{192}{EI}\right)\right] = -\frac{2.048}{EI}$$

$$\Delta_C = -\frac{11.264}{EI} - 2\left(-\frac{2048}{EI}\right) = -\frac{7.168}{EI} = -\frac{7.168}{200\times 250} = -0{,}143\,\text{m}$$

Soal Latihan (Chapter VIII)

- 8.10
- 8.11
- 8.12
- 8.13
- 8.14
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- 8.25