Universitas Pembangunan Jaya

Integrity, Professionalism, & Entrepreneurship

Mata Kuliah : Analisis Struktur
Kode : TSP – 202
SKS : 4 SKS

Deformasi Elastis Struktur Balok dan Portal

Pertemuan - 3

- Kemampuan Akhir yang Diharapkan
  - Mahasiswa dapat menganalisis deformasi struktur balok dengan metode integrasi ganda

- Sub Pokok Bahasan :
  - Pengertian Deformasi Lentur
  - Metode Integrasi Ganda

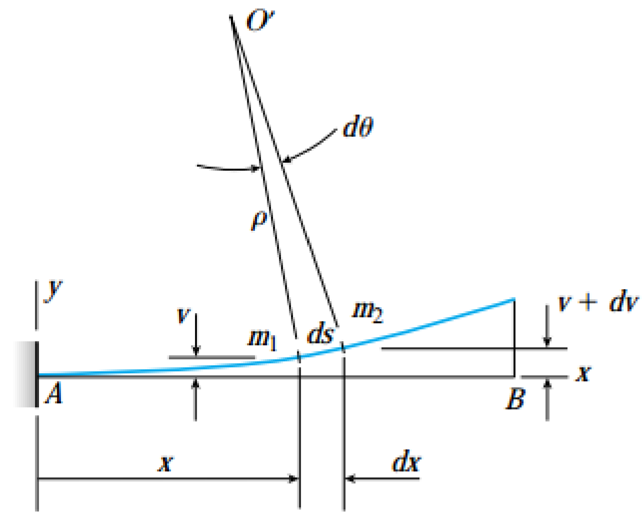
Elastic Beam Theory

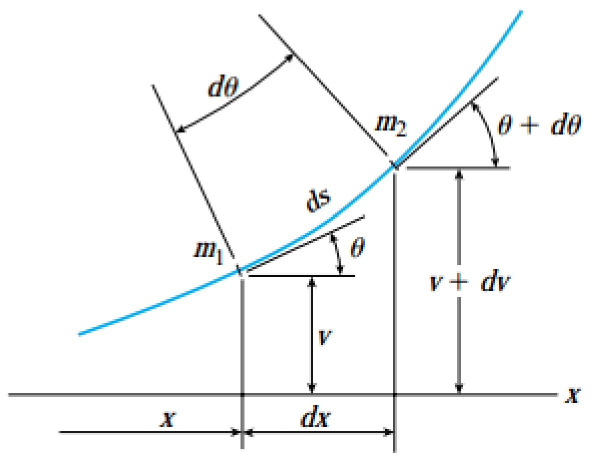
- Akibat beban yang bekerja, suatu balok akan berubah bentuk menjadi suatu lengkungan.
- Jika ρ adalah **radius kelengkungan**, dan κ adalah **kelengkungan**nya, maka hubungan keduanya adalah :

$$\kappa = \frac{1}{\rho} = \frac{d\theta}{ds} \approx \frac{d\theta}{dx} \qquad (1)$$

- Kemiringan kurva defleksi

$$\tan\theta \approx \theta = \frac{dv}{dx} \qquad (2)$$





- Dengan mengambil turunan terhadap x dari persamaan (2) :

$$\frac{d\theta}{dx} = \frac{d^2 v}{dx^2} \qquad \textbf{(3)}$$

- Gabungkan persamaan 1 dan 3 :

$$\kappa = \frac{1}{\rho} = \frac{d^2 v}{dx^2} \qquad \textbf{(4)}$$

- Jika bahan dari balok bersifat elastis linier dan mengikuti Hukum Hooke, maka :

$$\kappa = \frac{1}{\rho} = \frac{M}{EI} \qquad \textbf{(5) *}$$

*) lihat pada ilmu Mekanika Bahan

- Gabungkan persamaan (4) dan (5), maka akan diperoleh persamaan diferensial dasar untuk kurva defleksi suatu balok

$$\frac{d^2v}{dx^2} = \frac{M}{EI} \qquad \textbf{(6)}$$

- Setelah dapat menyatakan M sebagai fungsi dari posisi x, pengintegrasian secara berurutan dapat dilakukan untuk mendapatkan persamaan kurva deformasi elastis

$$v(x) = \iint \frac{M(x)}{EI}\,dx \qquad \textbf{(7)}$$

Perjanjian Tanda

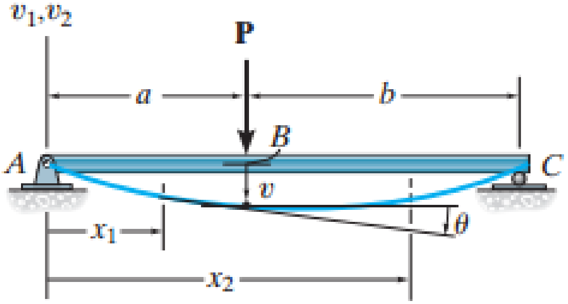
- Perpindahan vertikal *v* positif apabila berpindah ke arah atas, sedangkan sudut rotasi θ positif dihitung berlawanan arah jarum jam dari axis sumbu *x*.

Boundary Condition

- Perpindahan vertikal *v* = 0 pada setiap tumpuan, baik tumpuan rol, sendi, maupun jepit.
- Seperti juga dalam metode kerja maya, apabila terdapat lebih dari satu persamaan M sebagai fungsi x pada keseluruhan panjang balok, maka tidak dapat di terapkan sebuah integrasi tunggal.
- Berdasarkan prinsip kontinuitas, pada setiap area persamaan momen M, 0<x<a dan a<x<b, maka

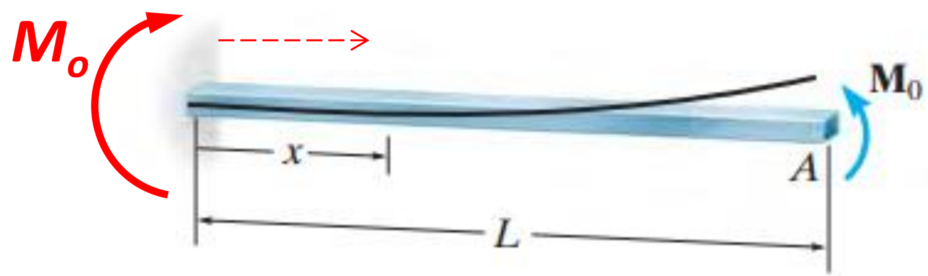
$$\theta_1(a) = \theta_2(a)$$

$$v_1(a) = v_2(a)$$



Example 1

The cantilevered beam shown in figure is subjected to a couple moment Mo at its end. Determine the equation of the elastic curve. Assume EI is constant



(a)

BC :

At x = 0 ; $dv/dx = 0$

At x = 0 ; $v = 0$

$$M_x = M_0$$

$$EI\frac{d^2v}{dx^2} = M_0$$

Using integration

$$EI\frac{dv}{dx} = M_0x + C_1 \qquad \textbf{(1)}$$

$$EIv = \frac{M_0x^2}{2} + C_1x + C_2 \qquad \textbf{(2)}$$

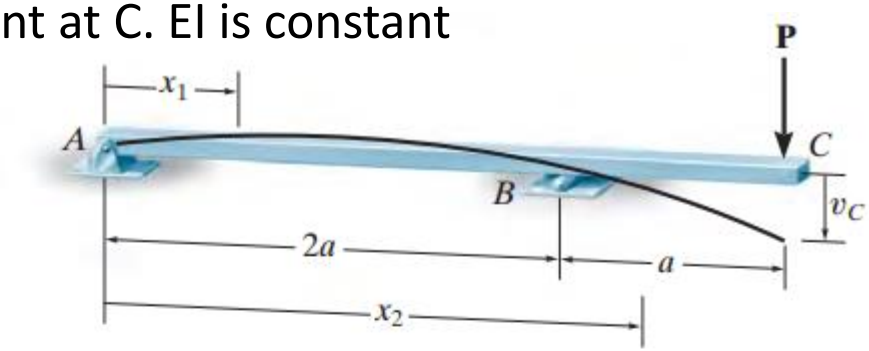
- Using the boundary conditions , we get $C_1 = C_2 = 0$. then Substituting these results into Eqs. (1) and (2) with we get

$$\frac{dv}{dx} = \theta = \frac{M_0 x}{EI}$$

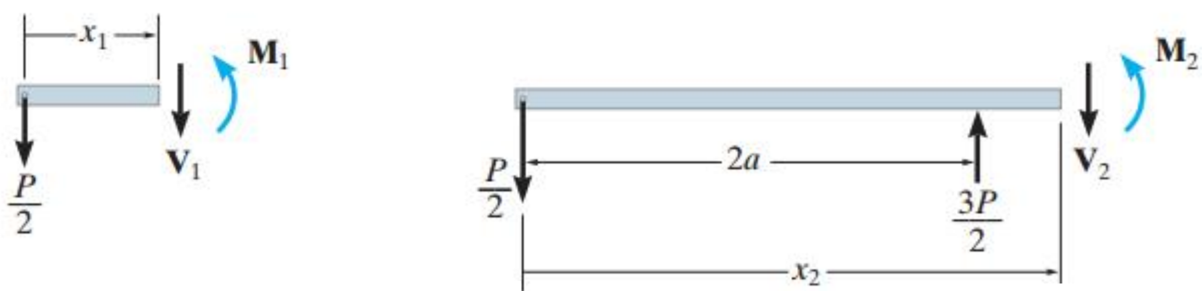
$$v = \frac{M_0 x^2}{2EI}$$

Example 2

The beam in figure is subjected to a load P at its end. Determine the displacement at C. EI is constant



(a)



$$M_1 = -\frac{P}{2}x_1 \qquad 0 \le x_1 \le 2a$$

$$M_2 = -\frac{P}{2}x_2 + \frac{3P}{2}(x_2 - 2a)$$

$$= Px_2 - 3Pa \qquad 2a \le x_2 \le 3a$$

for x_1,

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{P}{2}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{P}{4}x_1^2 + C_1$$

$$EIv_1 = -\frac{P}{12}x_1^3 + C_1x_1 + C_2$$

For x_2,

$$EI\frac{d^2v_2}{dx_2^2} = Px_2 - 3Pa$$

$$EI\frac{dv_2}{dx_2} = \frac{P}{2}x_2^2 - 3Pax_2 + C_3$$

$$EIv_2 = \frac{P}{6}x_2^3 - \frac{3}{2}Pax_2^2 + C_3x_2 + C_4$$

BC :

$v_1 = 0$ at $x_1 = 0$

$v_1 = 0$ at $x_1 = 2a$

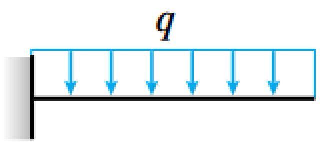
$v_2 = 0$ at $x_2 = 2a$

Persamaan Kontinuitas :

$dv_1/dx_1 = dv_2/dx_2$ at $x_1=x_2=2a$

Soal Latihan (Chapter VIII + Appendix G, Gere)

- **8.1**
- **8.2**
- **8.3**
- **8.4**
- **8.5**
- **8.6**
- **8.7**
- **8.8**
- **8.9**
- **Appendix G, Gere**

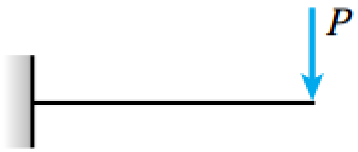


$$v = -\frac{qx^2}{24EI}(6L^2 - 4Lx + x^2) \qquad v' = -\frac{qx}{6EI}(3L^2 - 3Lx + x^2)$$

$$\delta_B = \frac{qL^4}{8EI} \qquad \theta_B = \frac{qL^3}{6EI}$$

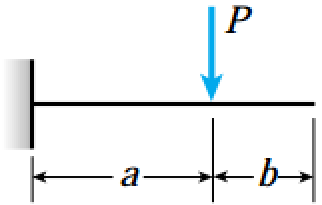
$$v = -\frac{M_0 x^2}{2EI} \qquad v' = -\frac{M_0 x}{EI}$$

$$\delta_B = \frac{M_0 L^2}{2EI} \qquad \theta_B = \frac{M_0 L}{EI}$$



$$v = -\frac{Px^2}{6EI}(3L - x) \qquad v' = -\frac{Px}{2EI}(2L - x)$$

$$\delta_B = \frac{PL^3}{3EI} \qquad \theta_B = \frac{PL^2}{2EI}$$

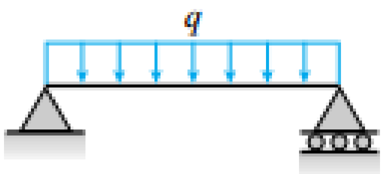


$$v = -\frac{Px^2}{6EI}(3a - x) \qquad v' = -\frac{Px}{2EI}(2a - x) \qquad (0 \le x \le a)$$

$$v = -\frac{Pa^2}{6EI}(3x - a) \qquad v' = -\frac{Pa^2}{2EI} \qquad (a \le x \le L)$$

$$\text{At } x = a: \quad v = -\frac{Pa^3}{3EI} \qquad v' = -\frac{Pa^2}{2EI}$$

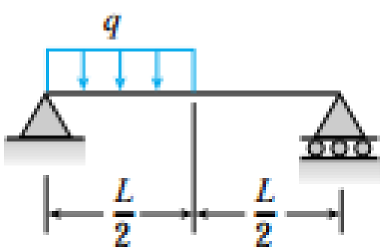
$$\delta_B = \frac{Pa^2}{6EI}(3L - a) \qquad \theta_B = \frac{Pa^2}{2EI}$$



$$v = -\frac{qx}{24EI}(L^3 - 2Lx^2 + x^3)$$

$$v' = -\frac{q}{24EI}(L^3 - 6Lx^2 - 4x^3)$$

$$\delta_C = \delta_{\max} = \frac{5qL^4}{384EI} \qquad \theta_A = \theta_B = \frac{qL^3}{24EI}$$



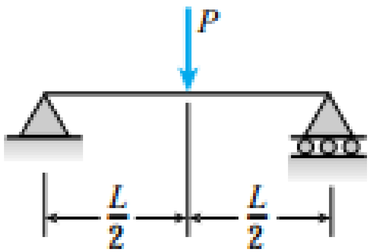
$$v = -\frac{qx}{384EI}(9L^3 - 24Lx^2 + 16x^3) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$v' = -\frac{q}{384EI}(9L^3 - 72Lx^2 + 64x^3) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$v = -\frac{qL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3) \qquad \left(\frac{L}{2} \le x \le L\right)$$

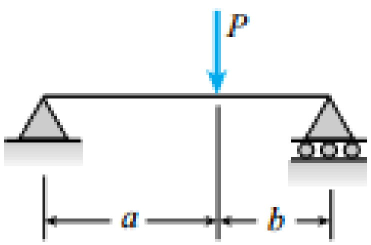
$$v' = -\frac{qL}{384EI}(24x^2 - 48Lx + 17L^2) \qquad \left(\frac{L}{2} \le x \le L\right)$$

$$\delta_C = \frac{5qL^4}{768EI} \qquad \theta_A = \frac{3qL^3}{128EI} \qquad \theta_B = \frac{7qL^3}{384EI}$$



$$v = -\frac{Px}{48EI}(3L^2 - 4x^2) \qquad v' = -\frac{P}{16EI}(L^2 - 4x^2) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$\delta_C = \delta_{max} = \frac{PL^3}{48EI} \qquad \theta_A = \theta_B = \frac{PL^2}{16EI}$$

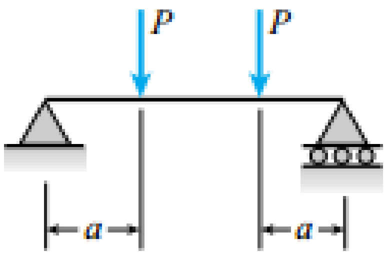


$$v = -\frac{Pbx}{6LEI}(L^2 - b^2 - x^2) \qquad v' = -\frac{Pb}{6LEI}(L^2 - b^2 - 3x^2) \qquad (0 \le x \le a)$$

$$\theta_A = \frac{Pab(L + b)}{6LEI} \qquad \theta_B = \frac{Pab(L + a)}{6LEI}$$

$$\text{If } a \ge b, \quad \delta_C = \frac{Pb(3L^2 - 4b^2)}{48EI} \qquad \text{If } a \le b, \quad \delta_C = \frac{Pa(3L^2 - 4a^2)}{48EI}$$

$$\text{If } a \ge b, \quad x_1 = \sqrt{\frac{L^2 - b^2}{3}} \quad \text{and} \quad \delta_{max} = \frac{Pb(L^2 - b^2)^{3/2}}{}$$



$$v = -\frac{Px}{6EI}(3aL - 3a^2 - x^2) \qquad v' = -\frac{P}{2EI}(aL - a^2 - x^2) \qquad (0 \le x \le a)$$

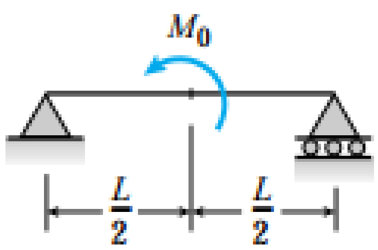
$$v = -\frac{Pa}{6EI}(3Lx - 3x^2 - a^2) \qquad v' = -\frac{Pa}{2EI}(L - 2x) \qquad (a \le x \le L - a)$$

$$\delta_C = \delta_{max} = \frac{Pa}{24EI}(3L^2 - 4a^2) \qquad \theta_A = \theta_B = \frac{Pa(L - a)}{2EI}$$

$$v = -\frac{M_0 x}{6LEI}(2L^2 - 3Lx + x^2) \qquad v' = -\frac{M_0}{6LEI}(2L^2 - 6Lx + 3x^2)$$

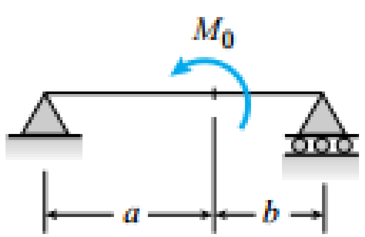
$$\delta_C = \frac{M_0 L^2}{16EI} \qquad \theta_A = \frac{M_0 L}{3EI} \qquad \theta_B = \frac{M_0 L}{6EI}$$

$$x_1 = L\left(1 - \frac{\sqrt{3}}{3}\right) \quad \text{and} \quad \delta_{max} = \frac{M_0 L^2}{9\sqrt{3}EI}$$



$$v = -\frac{M_0 x}{24LEI}(L^2 - 4x^2) \qquad v' = -\frac{M_0}{24LEI}(L^2 - 12x^2) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$\delta_C = 0 \qquad \theta_A = \frac{M_0 L}{24EI} \qquad \theta_B = -\frac{M_0 L}{24EI}$$



$$v = -\frac{M_0 x}{6LEI}(6aL - 3a^2 - 2L^2 - x^2) \qquad (0 \le x \le a)$$

$$v' = -\frac{M_0}{6LEI}(6aL - 3a^2 - 2L^2 - 3x^2) \qquad (0 \le x \le a)$$

$$\text{At } x = a: \quad v = -\frac{M_0 ab}{3LEI}(2a - L) \qquad v' = -\frac{M_0}{3LEI}(3aL - 3a^2 - L^2)$$

$$\theta_A = \frac{M_0}{6LEI}(6aL - 3a^2 - 2L^2) \qquad \theta_B = \frac{M_0}{6LEI}(3a^2 - L^2)$$