Universitas Pembangunan Jaya

Integrity, Professionalism, & Entrepreneurship

Mata Kuliah : Statika & Mekanika Bahan
Kode : CIV – 102
SKS : 4 SKS

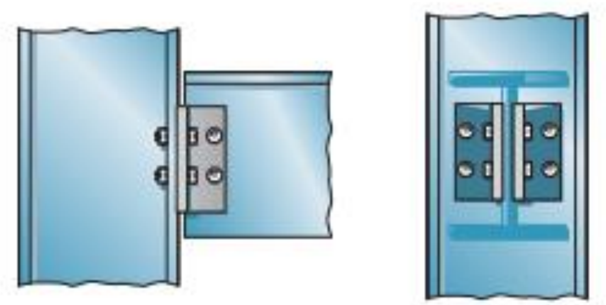
Struktur Statis Tertentu : Balok

Pertemuan – 2 s/d 7

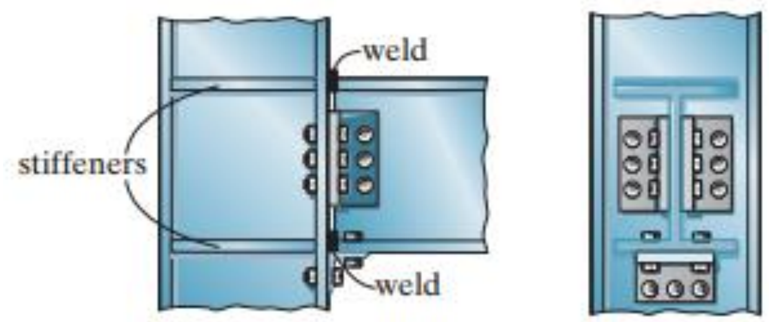
- Kemampuan akhir yang diharapkan
  - Mahasiswa dapat menganalisis gaya dalam momen, lintang dan netral pada struktur balok sederhana
- Bahan Kajian (Materi Ajar)
  - Jenis Tumpuan dan Sifatnya
  - Struktur Balok Sederhana
  - Struktur 3 sendi
  - Analisa reaksi perletakan
  - Gaya dalam momen, Lintang, Normal

Support Connections

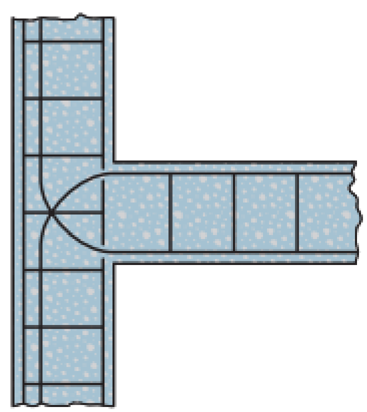
- Structural members are joined together in various ways depending on the intent of the designer.
- The three types of joints most often specified are the **pin connection**, the **roller support**, and **the fixed joint**.



typical "pin-supported" connection (metal)
(a)



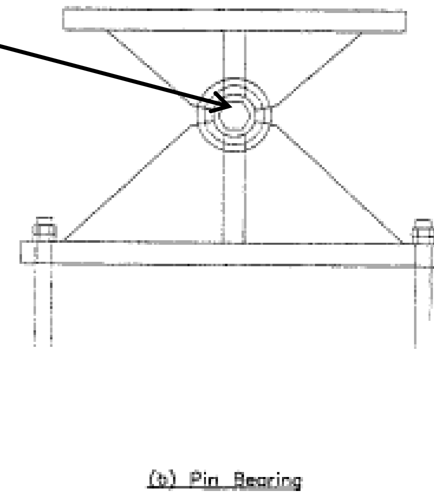
typical "fixed-supported" connection (metal)
(b)



typical "fixed-supported" connection (concrete)

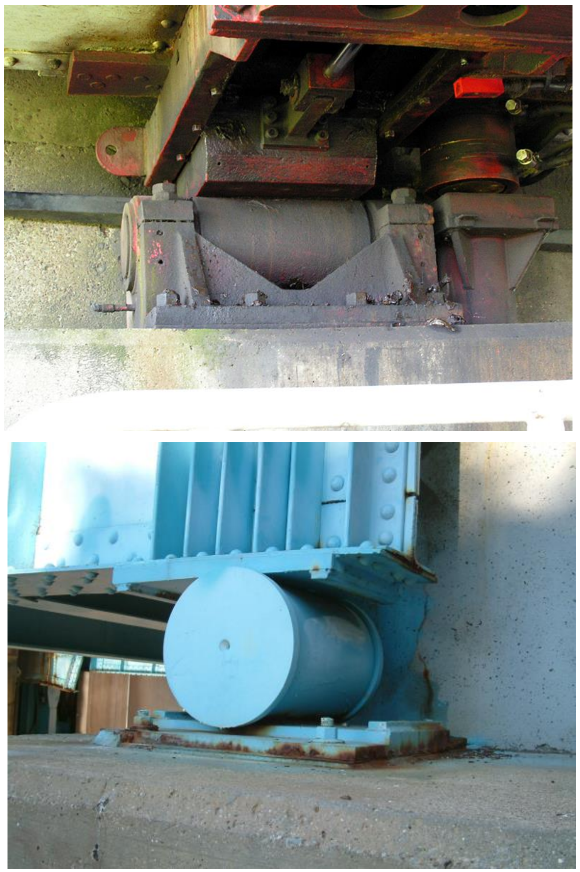
Pin Bearing

Steel Pin



- **Rotational Movement is allowed**
- **Lateral and Translational Movements are Restricted**

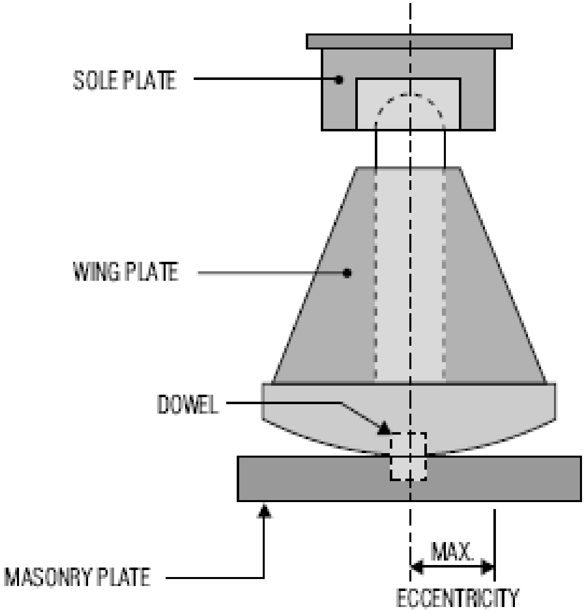
Roller Type Bearings



Single Roller Bearing

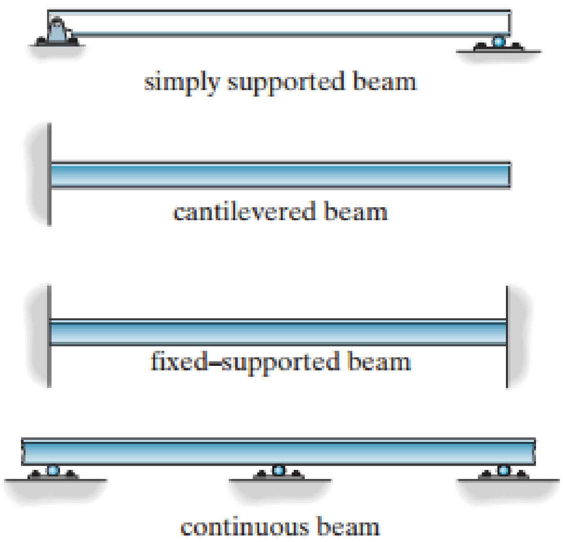
Multiple Roller Bearing

Rocker Type Bearing

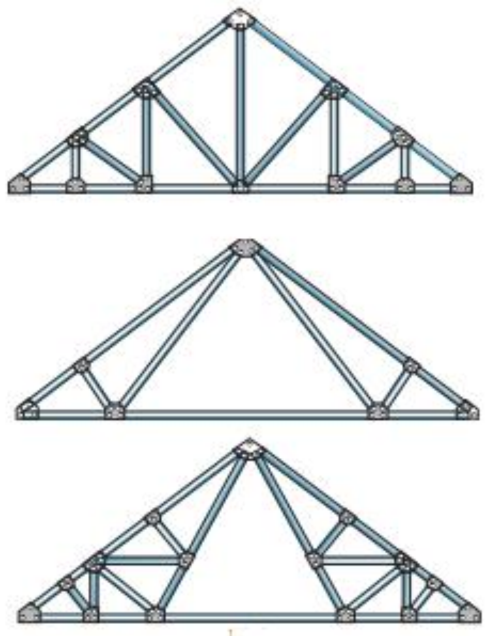


- A rocker bearing is a type of expansion bearing that comes in a great variety.
- It typically consists of a pin at the top that facilitates rotations, and a curved surface at the bottom that accommodates the translational movements
- Rocker and pin bearings are primarily used in steel bridges.

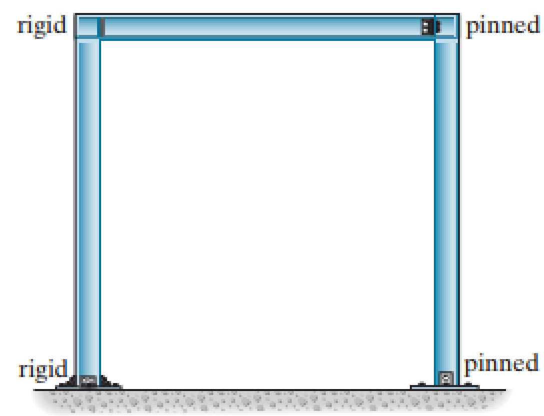
- Classification of Structures



BEAM

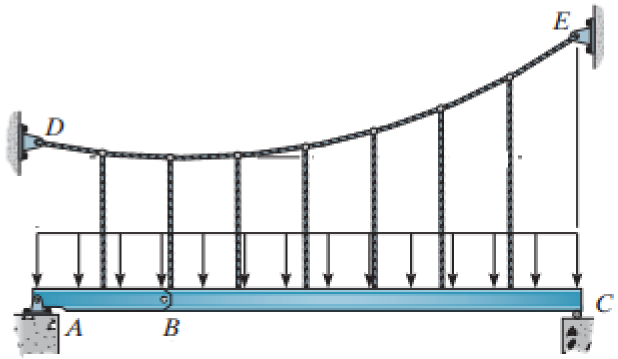


2D TRUSS

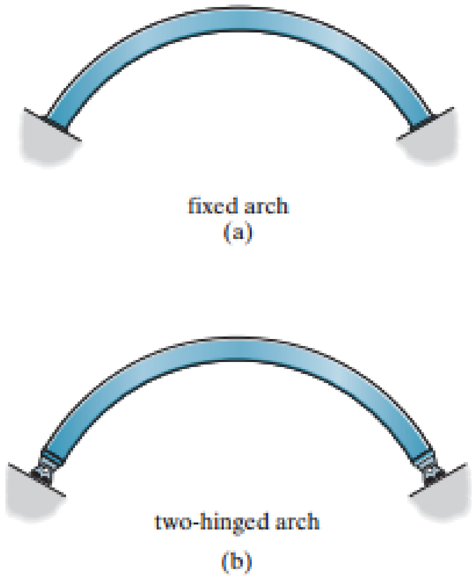


2D FRAME

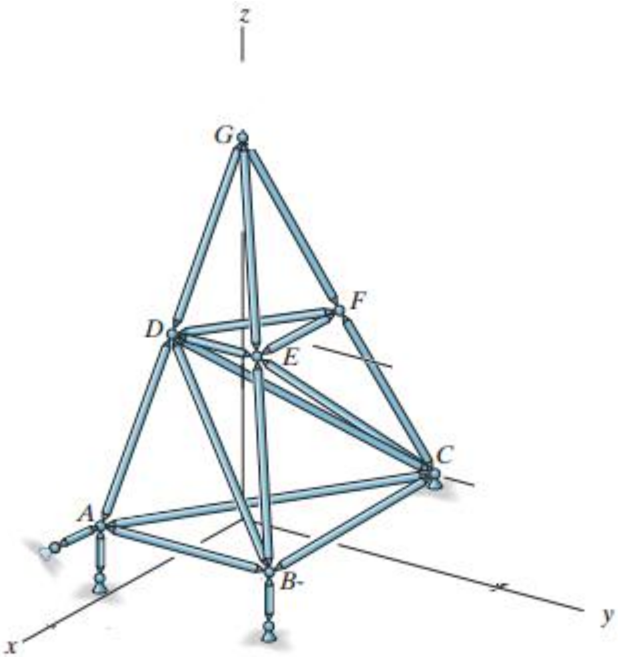
- Classification of Structures



CABLE

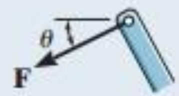
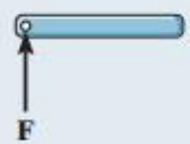


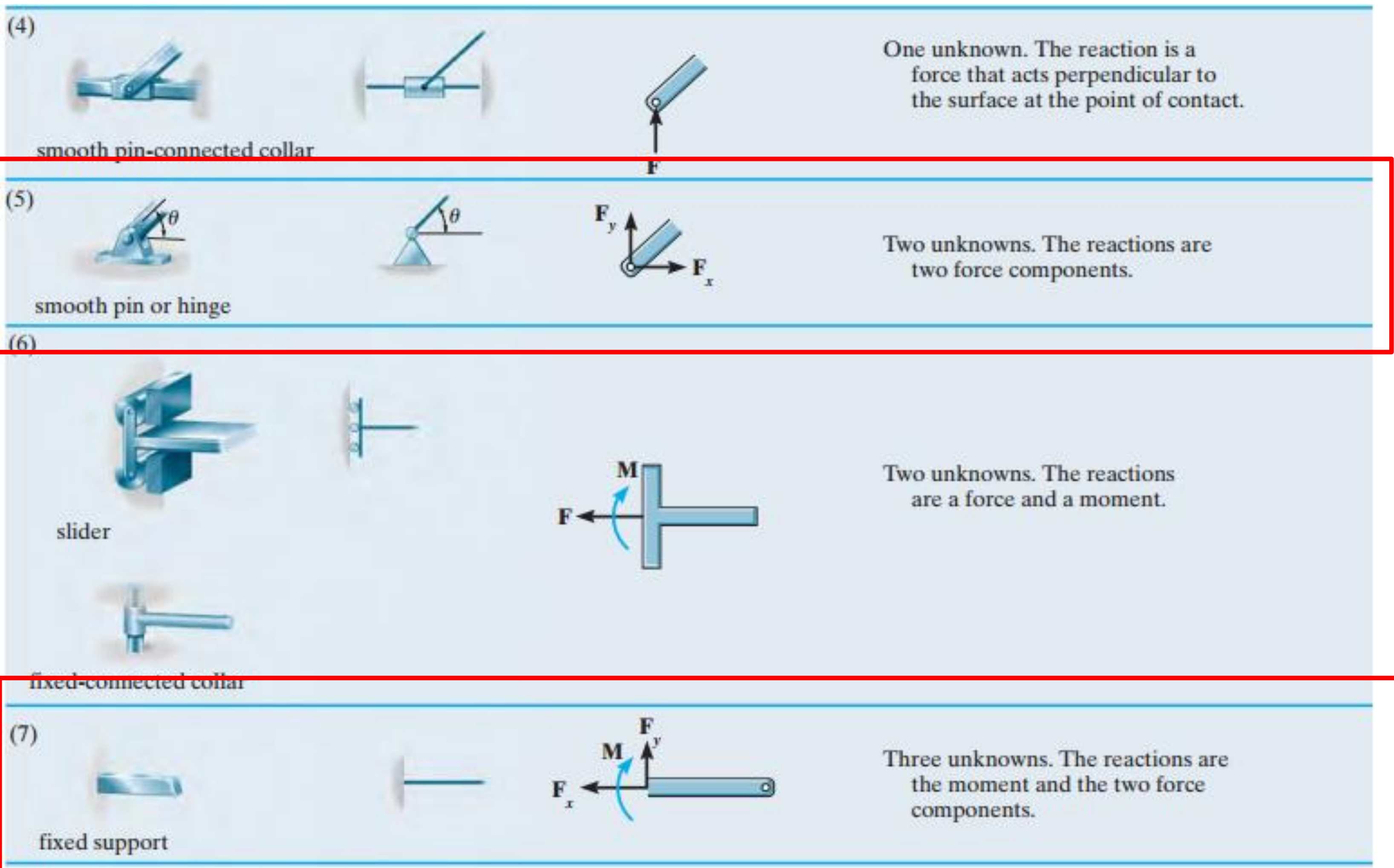
ARCH



3D TRUSS

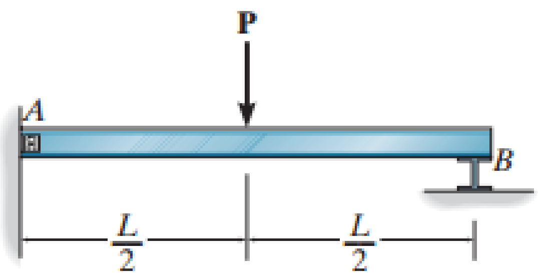
TABLE 2–1 Supports for Coplanar Structures

Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(1) 			One unknown. The reaction is a force that acts in the direction of the cable or link.
(2) 			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.

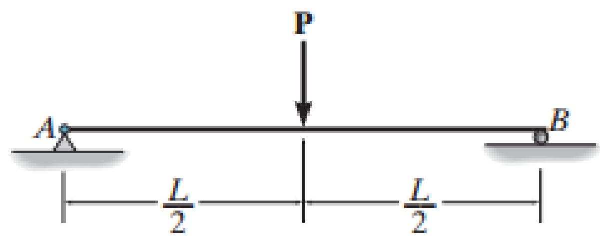


	Type of connection	Reaction	Number of unknowns
(4)	smooth pin-connected collar	F	One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(5)	smooth pin or hinge (θ)	F_y, F_x	Two unknowns. The reactions are two force components.
(6)	slider; fixed-connected collar	M, F	Two unknowns. The reactions are a force and a moment.
(7)	fixed support	F_y, M, F_x	Three unknowns. The reactions are the moment and the two force components.

Idealized Structure



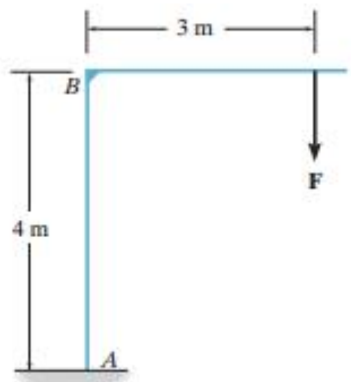
actual beam
(a)



idealized beam
(b)

actual structure

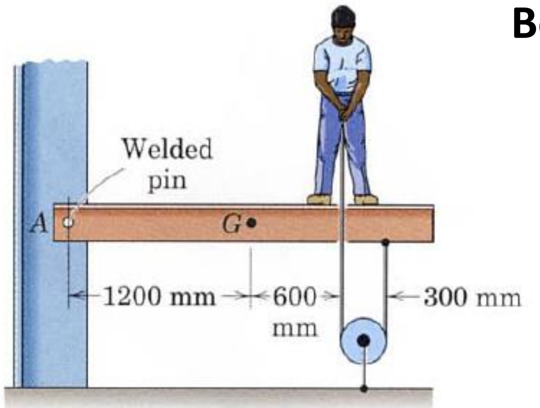
(a)



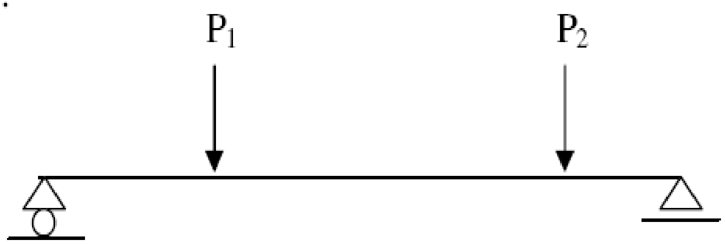
idealized structure

(b)

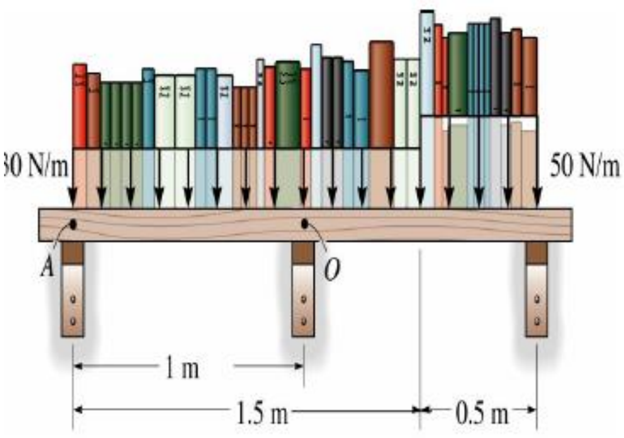
Loading

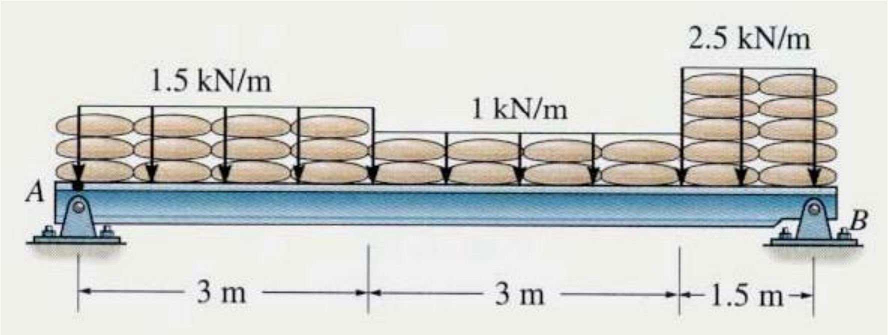


Beban terpusat



Beban terdistribusi

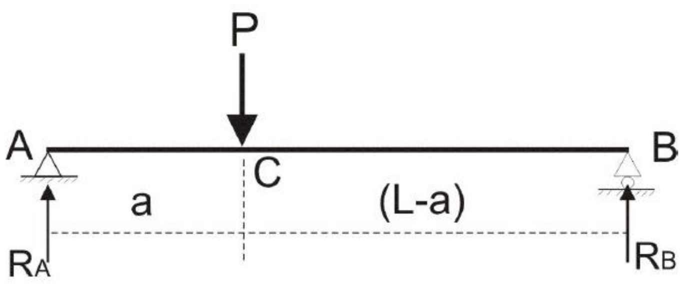


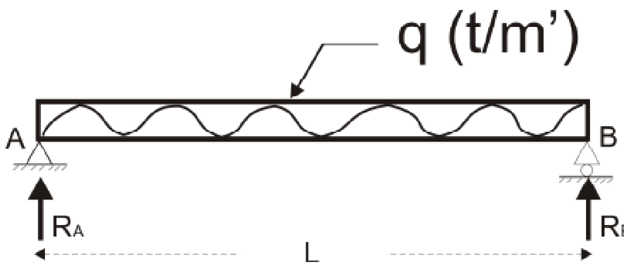


Balok Sederhana (*Simple Beam*)

Beam-structural member designed to support loads applied at various points along its length.

Balok sederhana (simple beam): Kedua ujungnya ditumpu oleh jenis tumpuan sendi dan Rol





- **Kesetimbangan**

  Keadaan dari suatu benda dimana resultante dari gaya-gaya yang bekerja sama dengan Nol

- persamaan kesetimbangan dapat dituliskan sebagai :

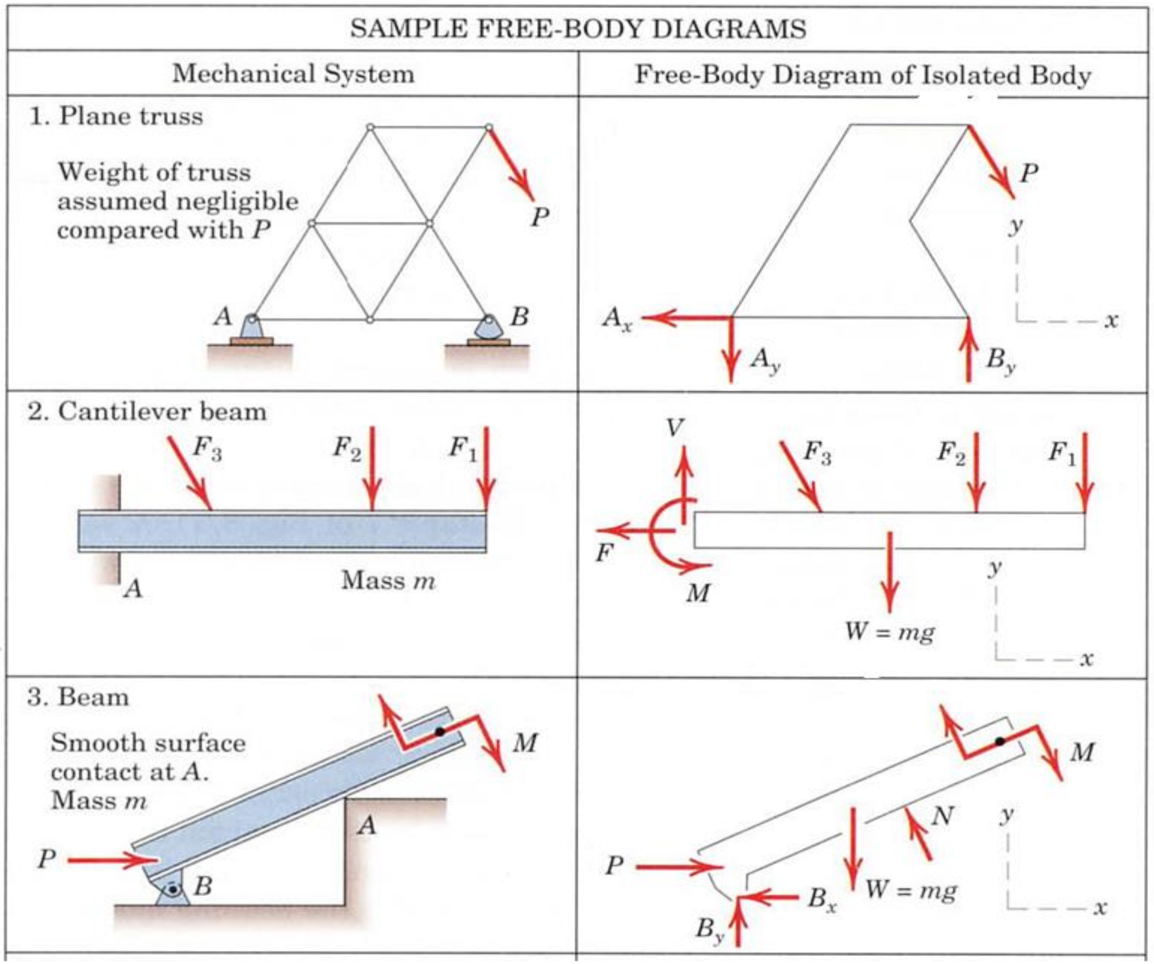
$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M = 0$$

- Jika tidak memenuhi persyaratan tersebut, maka benda dikatakan dalam keadaan tidak seimbang (mengalami gerak translasi, rotasi, atau kombinasi keduanya)

FreeBody Diagram

- Gaya reaksi dengan arah dan besarnya menyeimbangkan gaya-gaya yang beraksi pada benda
- Free Body Diagram (diagram benda bebas) akan mempermudah untuk menentukan besar dan arah gaya reaksi tersebut.

the free-body diagram is the most important single step in the solution of problems in mechanics.


SAMPLE FREE-BODY DIAGRAMS
Mechanical System
Free-Body Diagram of Isolated Body
1. Plane truss
Weight of truss assumed negligible compared with P
P
A
B
P
y
x
A_x
A_y
B_y
2. Cantilever beam
F_3
F_2
F_1
A
Mass m
V
F_3
F_2
F_1
F
M
W = mg
y
x
3. Beam
Smooth surface contact at A. Mass m
M
A
P
B
M
N
y
P
B_x
W = mg
B_y
x

Determinacy

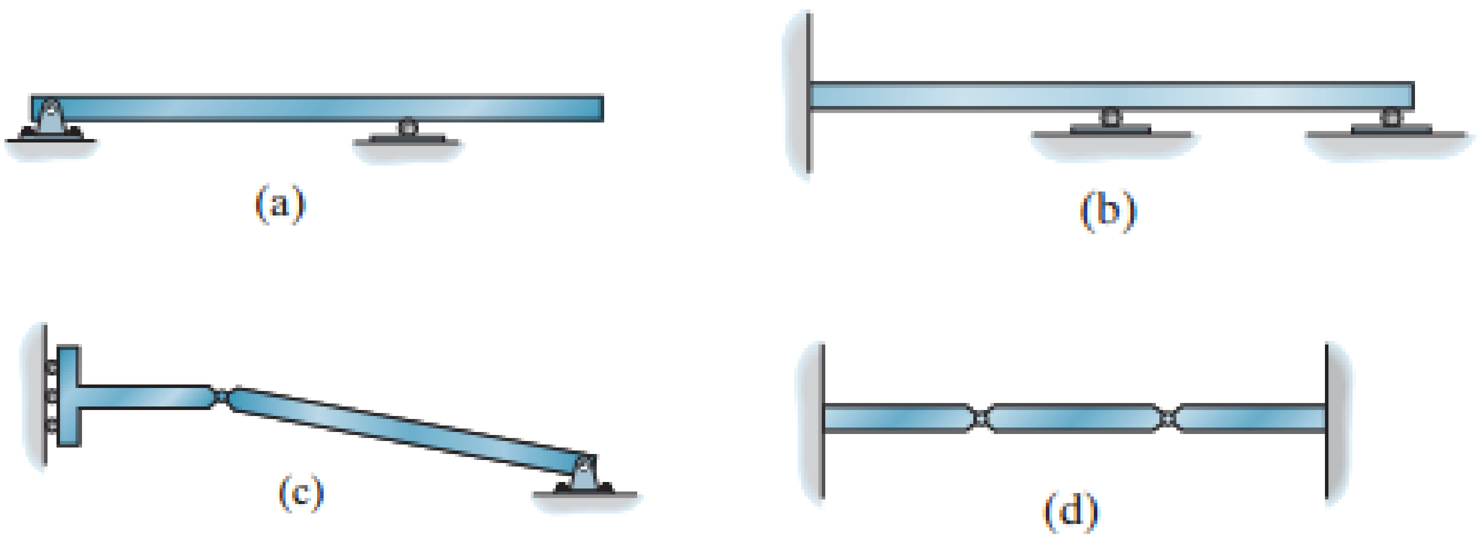
- The equilibrium equations provide both the *necessary and sufficient* conditions for equilibrium.
- When all the forces in a structure can be determined strictly from these equations, the structure is referred to as ***statically determinate***.
- Structures having more unknown forces than available equilibrium equations are called ***statically indeterminate***.

- For a coplanar structure there are at most *three* equilibrium equations for each part, so that if there is a total of *n* parts and *r* force and moment reaction components

$$r = 3n, \text{ statically determinate}$$
$$r > 3n, \text{ statically indeterminate}$$

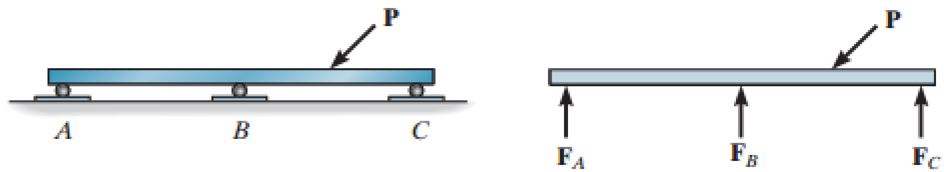
Example 1

- Classify each of the beams shown in Fig. *a* through *d* as statically determinate or statically indeterminate.

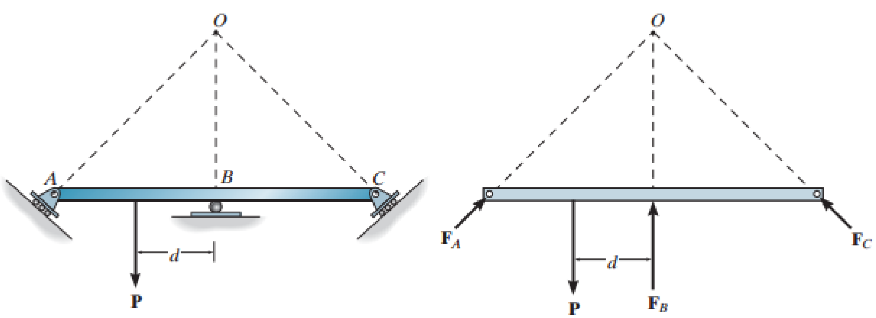


- Stability

$r < 3n$	unstable
$r \geq 3n$	unstable if member reactions are concurrent or parallel or some of the components form a collapsible mechanism



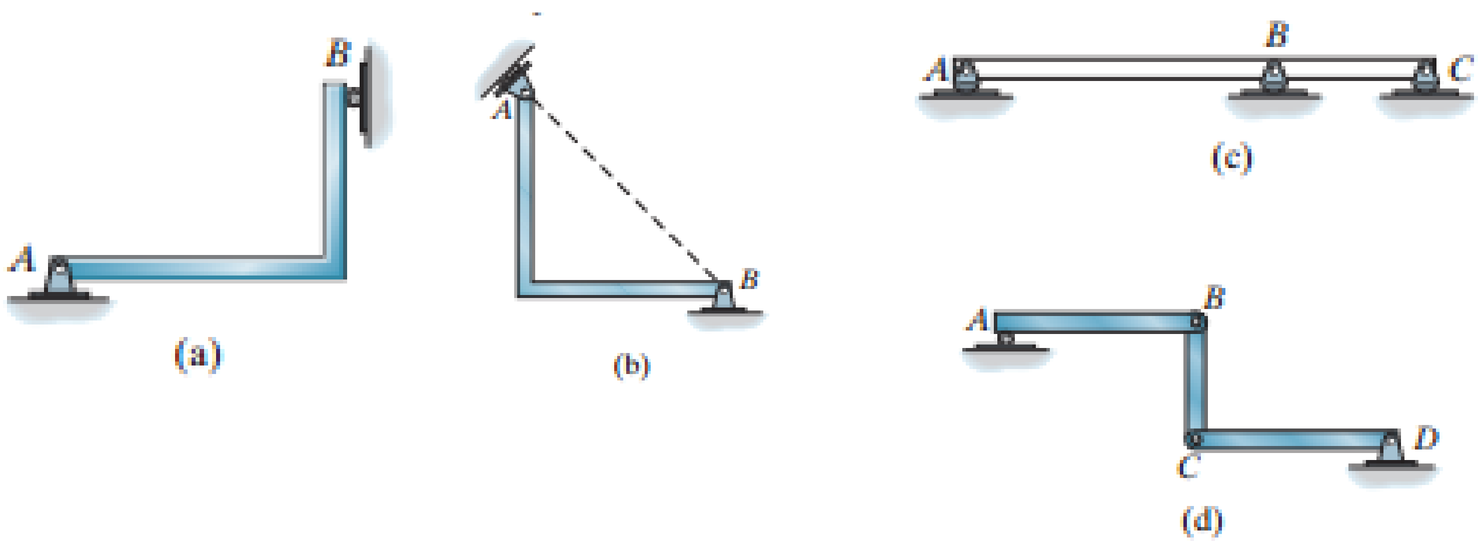
parallel reactions



concurrent reactions

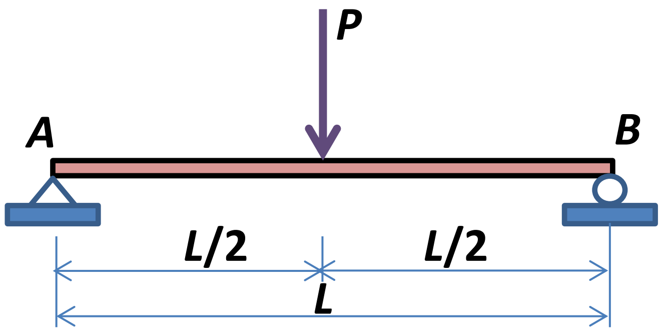
Example 2

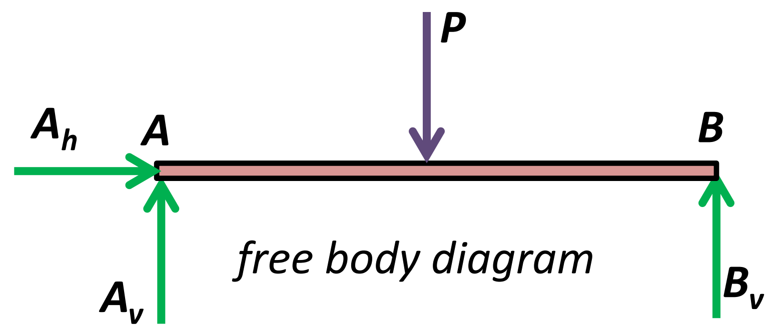
- Classify each of the structures in Fig. *a* through *d* as stable or unstable.



Example 3

- Determine the reaction of the beam





$\Sigma M_A = 0$

$+P(L/2) - B_v(L) = 0$

$B_v(L) = P(L/2)$

$B_v = P/2$

$\Sigma M_B = 0$

$-P(L/2) + A_v(L) = 0$

$A_v(L) = P(L/2)$

$A_v = P/2$

$\Sigma F_y = 0$

$A_v - P + B_v = 0$

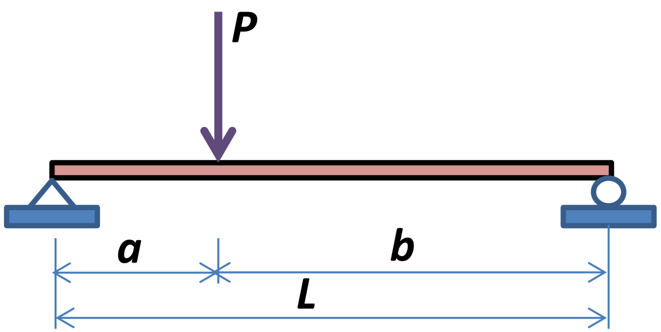
$P/2 - P - P/2 = 0$ (OK!)

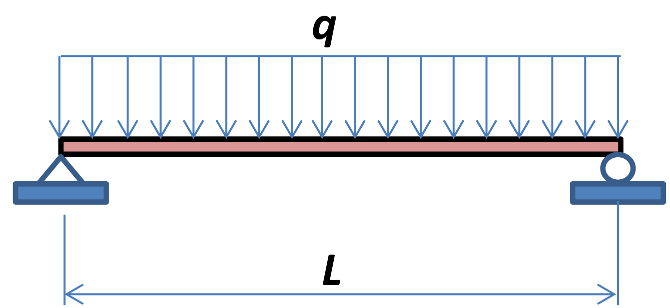
$\Sigma F_x = 0$

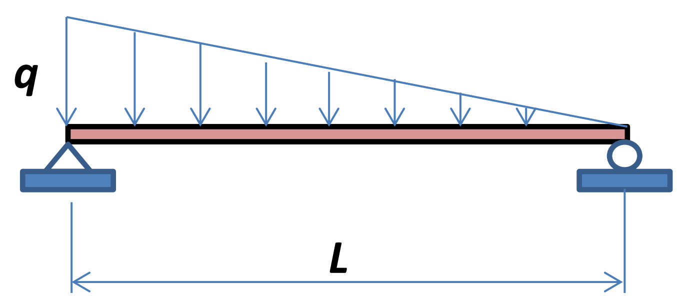
$A_h = 0$

Example 4

- Determine the reaction of the beam

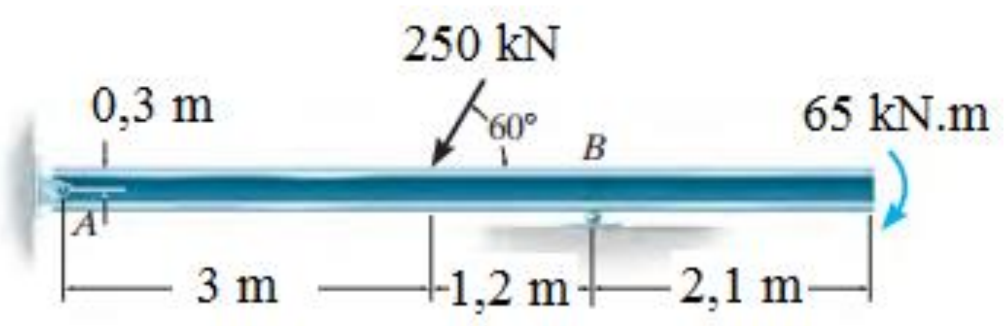






Example 4

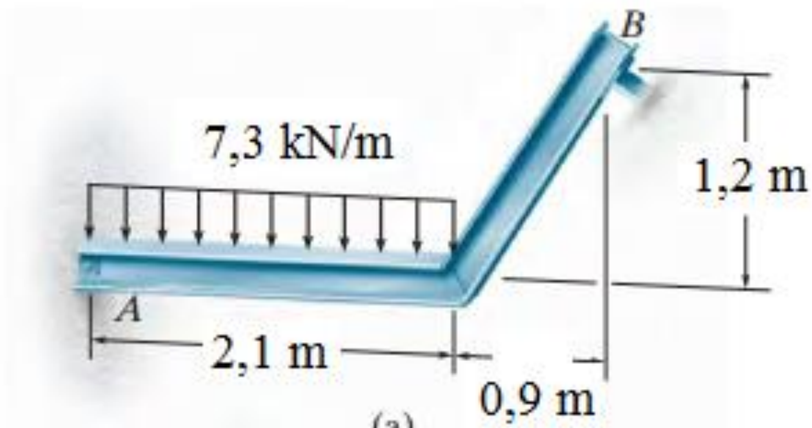
- Determine the reaction on the beam. A is pin and B is a roller support



(a)

Example 5

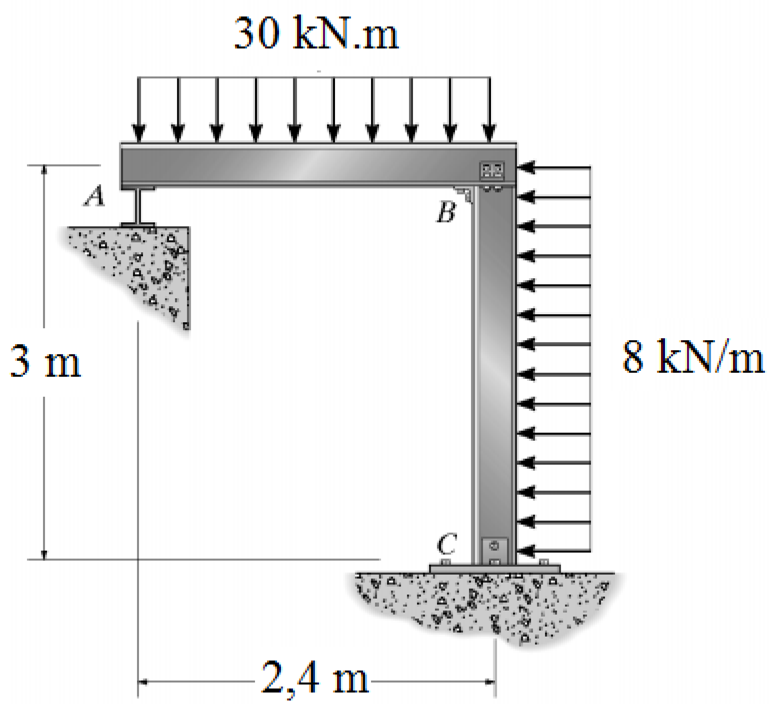
- Determine the reaction on the beam



(a)

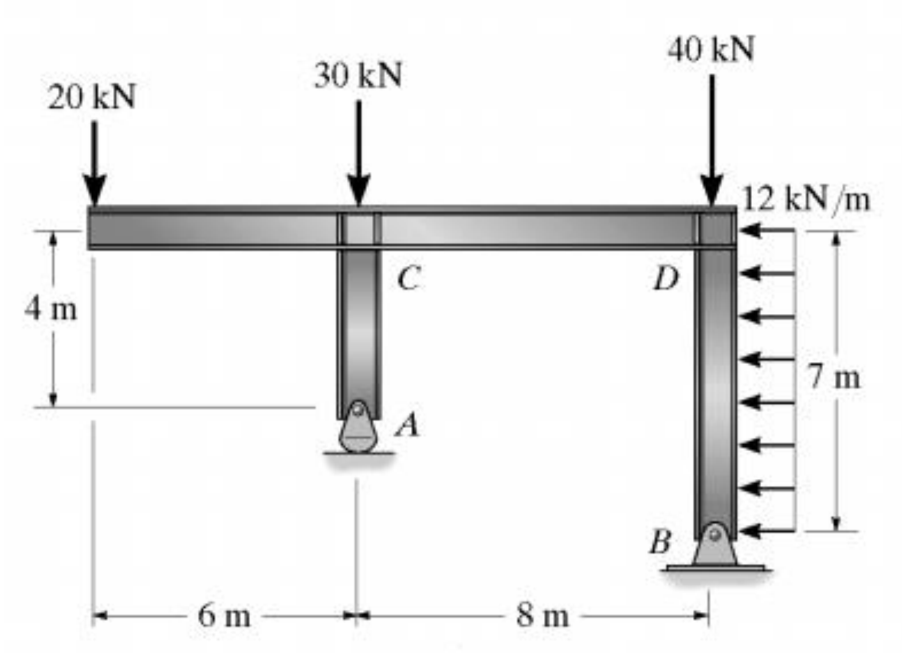
Example 6

- Determine the reactions at the supports A and C. Assume the support at A is a roller, B is a fixed-connected joint and C is a pin.



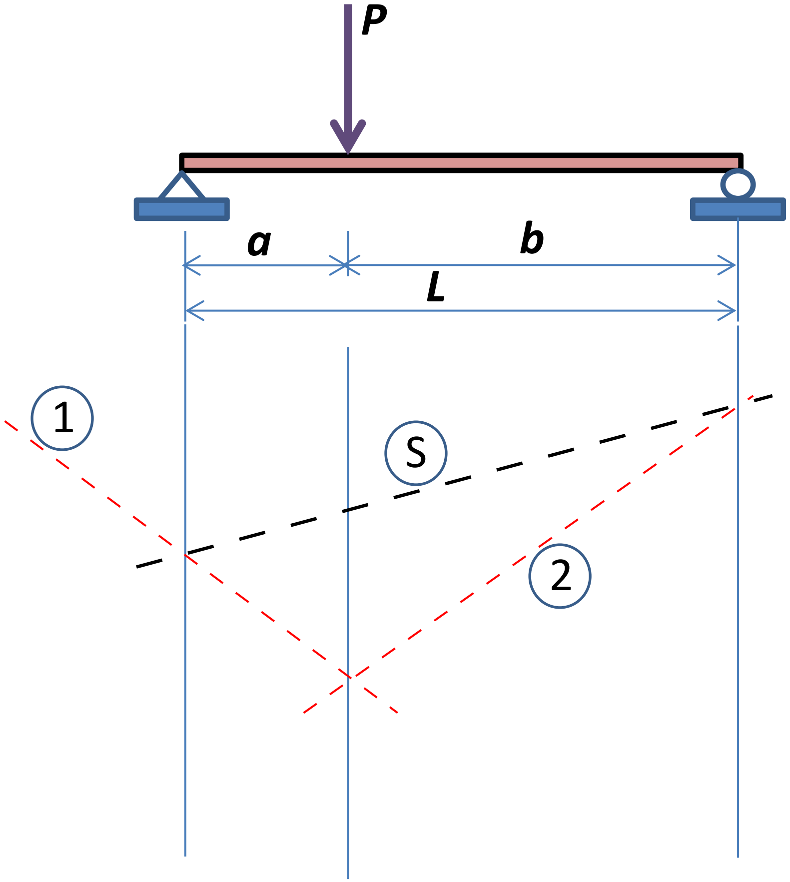
Example 7

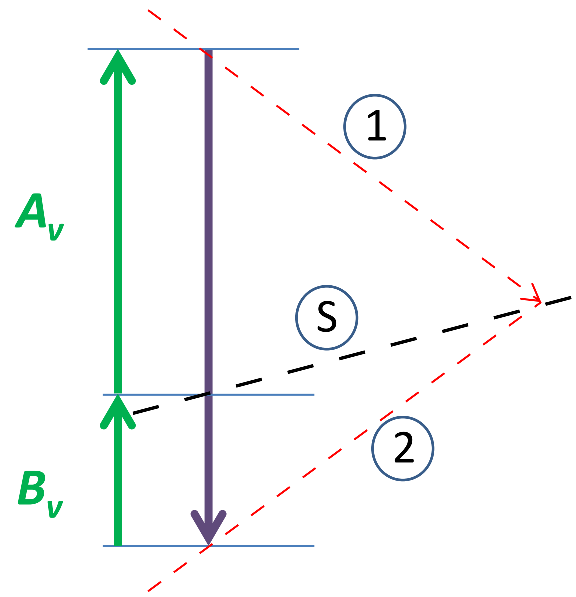
- Determine the horizontal and vertical components of reaction at the supports A (roller) and B (pin). The joints at C and D are fixed connections



- Graphical Method

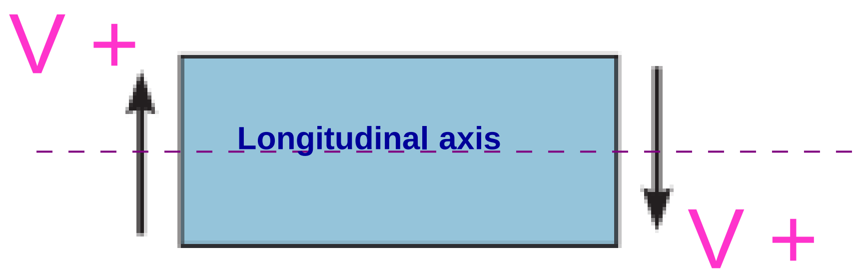
Panjang, dan beban harus diskalakan
misal : 1m = 1cm; 10 kN = 1 cm





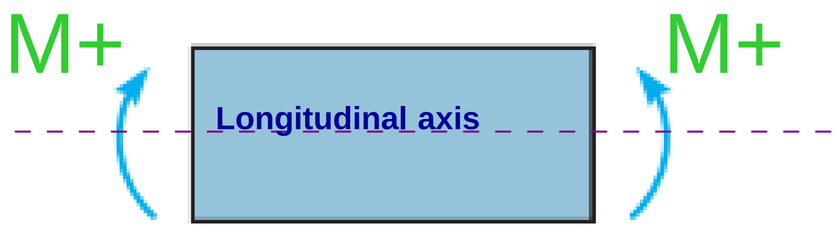
Shear Force

The algebraic sum of the components acting **transverse** to the axis of the beam of all the loads and reactions applied to the portion of the beam on either side of that cross section



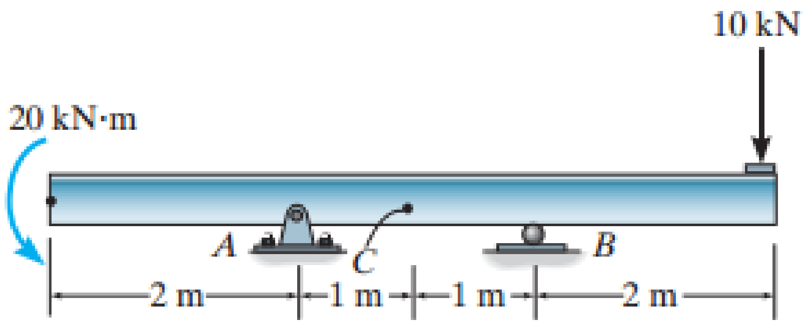
Bending Moment

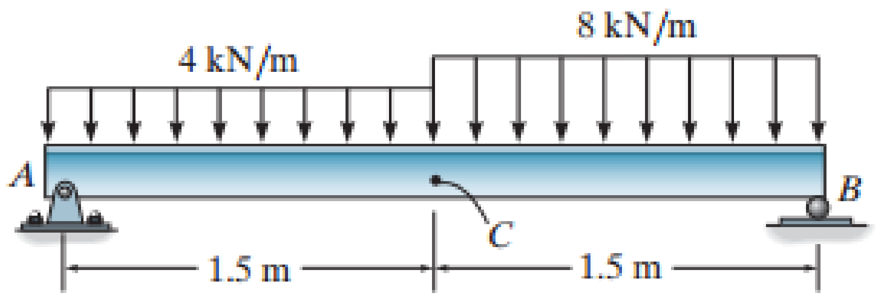
The algebraic sum of the moments, taken about an axis (which is **normal** to the plane of loading), passing through the centroid of the cross section of all the loads and reactions applied to the portion of the beam on either side of that cross section.



- # Example 1

Determine the internal normal force, shear force and bending moment acting at point C in the beam.





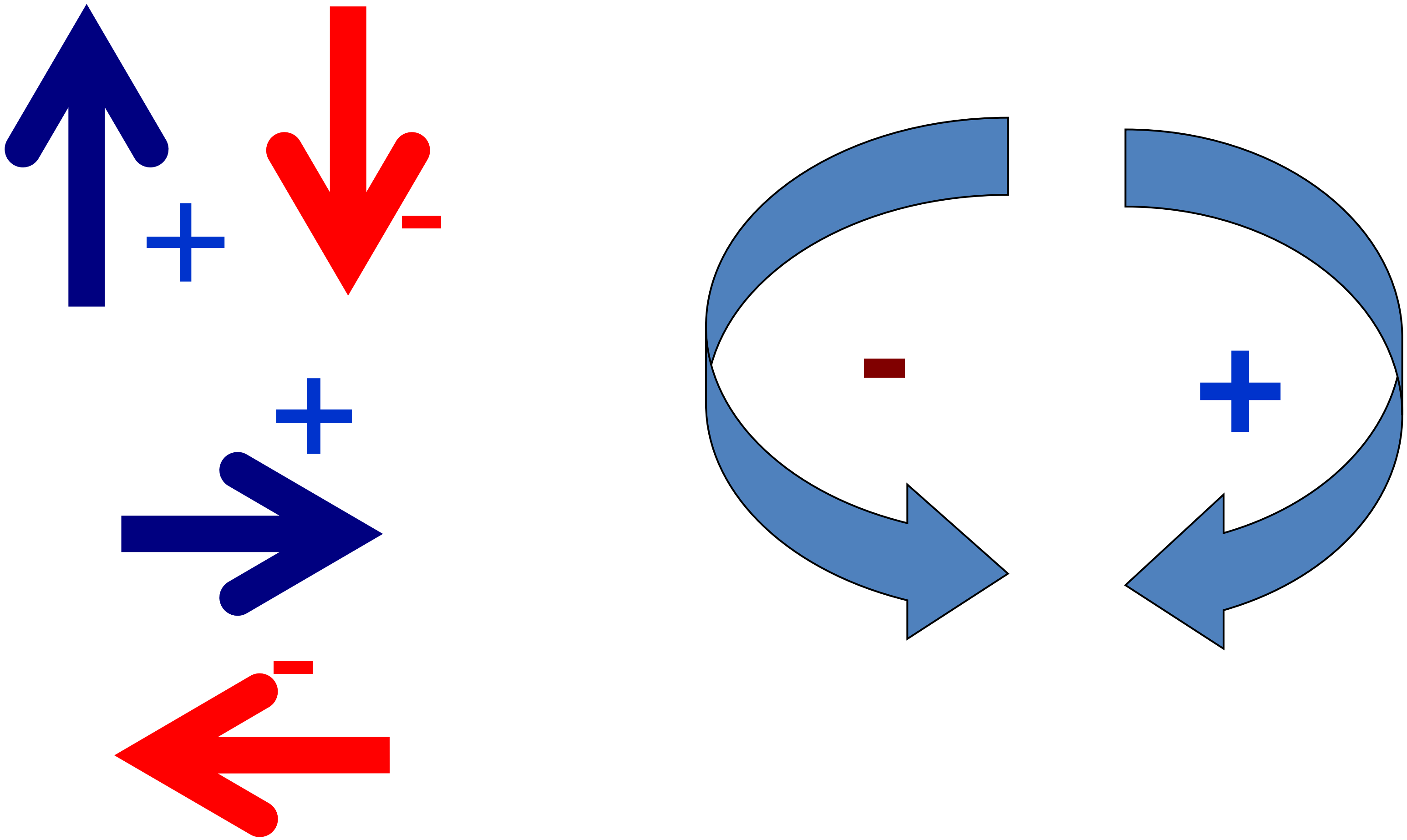
Shear and Moment Functions

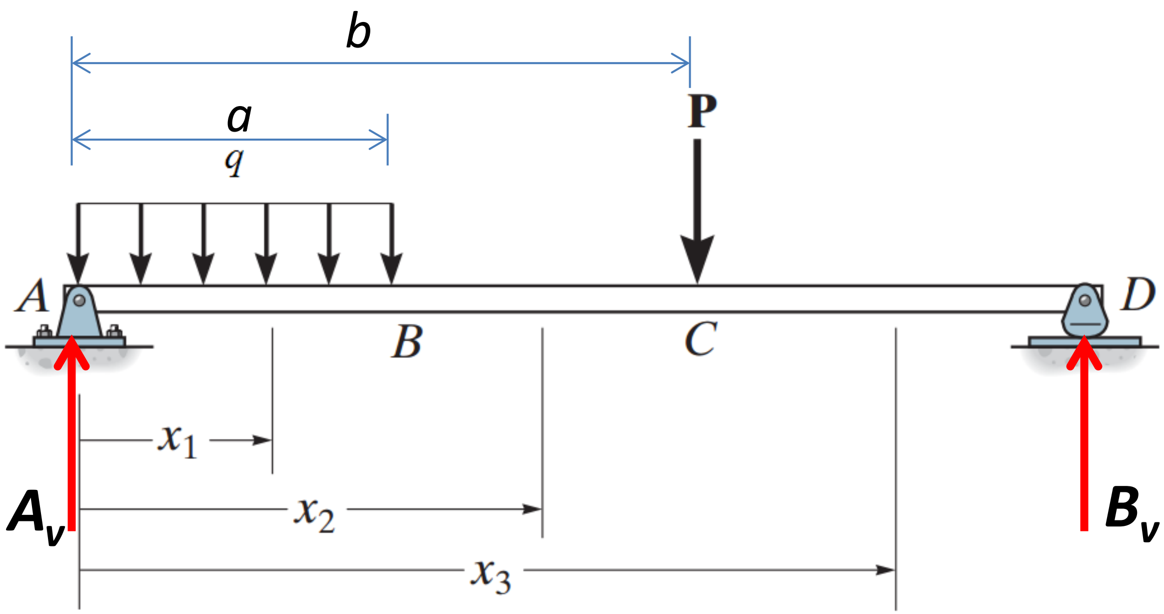
- The design of a beam requires a detailed knowledge of the *variations* of the internal shear force *V* and moment *M* acting at each point along the axis of the beam.
- The variations of *V* and *M* as a function of the position *x* of an arbitrary point along the beam's axis can be obtained by using the method of sections
- shear and moment functions must be determined for each region of the beam located *between* any two discontinuities of loading

Procedure for Analysis

- Determine the support reactions on the beam
- Specify separate coordinates *x* and associated origins
- Section the beam perpendicular to its axis at each distance *x*, draw free-body diagram
- V_x and M_x obtained from equilibrium equation
- The results can be checked by noting that ***dM/dx* = V** and ***dV/dx* = *q***, where q is positive when it acts upward, away from the beam

- Sign Convention

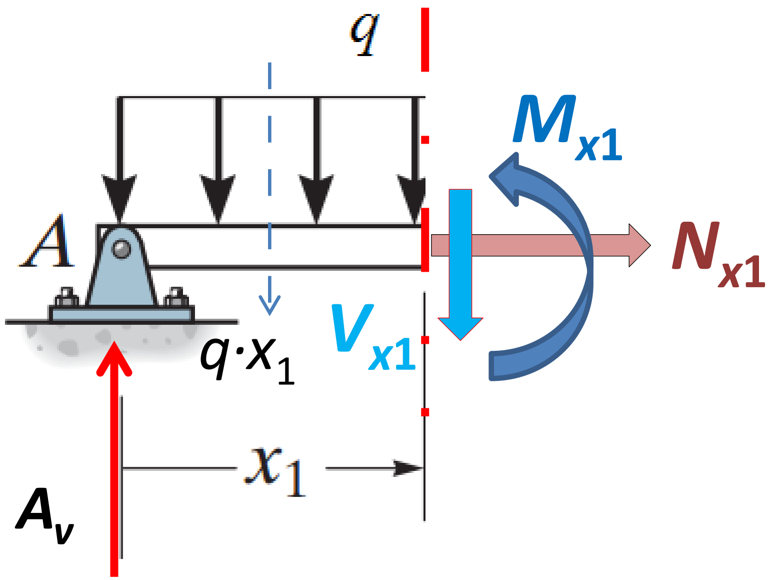




From equilibrium equation :

$\Sigma M_A = 0, \Sigma M_B = 0, \Sigma F_y = 0$

Determine A_v and B_v



For region A-B :

$\Sigma F_y = 0$

$+A_v \square\, q \cdot x_1 - V_{x1} = 0$

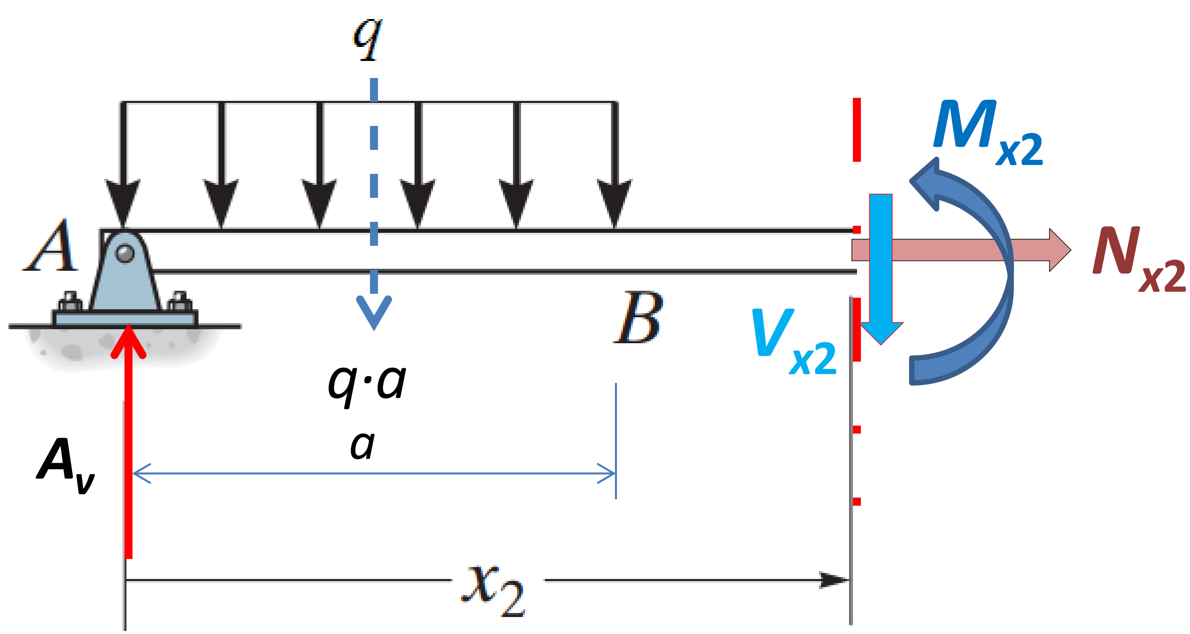
$V_{x1} = A_v \square\, q \cdot x_1$

$\Sigma M_{x1} = 0$

$+A_v \cdot x_1 - ½q \cdot x_1^2 - M_{x1} = 0$

$M_{x1} = A_v \cdot x_1 - ½q \cdot x_1^2$

Check for dM_{x1}/dx_1 !



For region B-C :

$\Sigma F_y = 0$

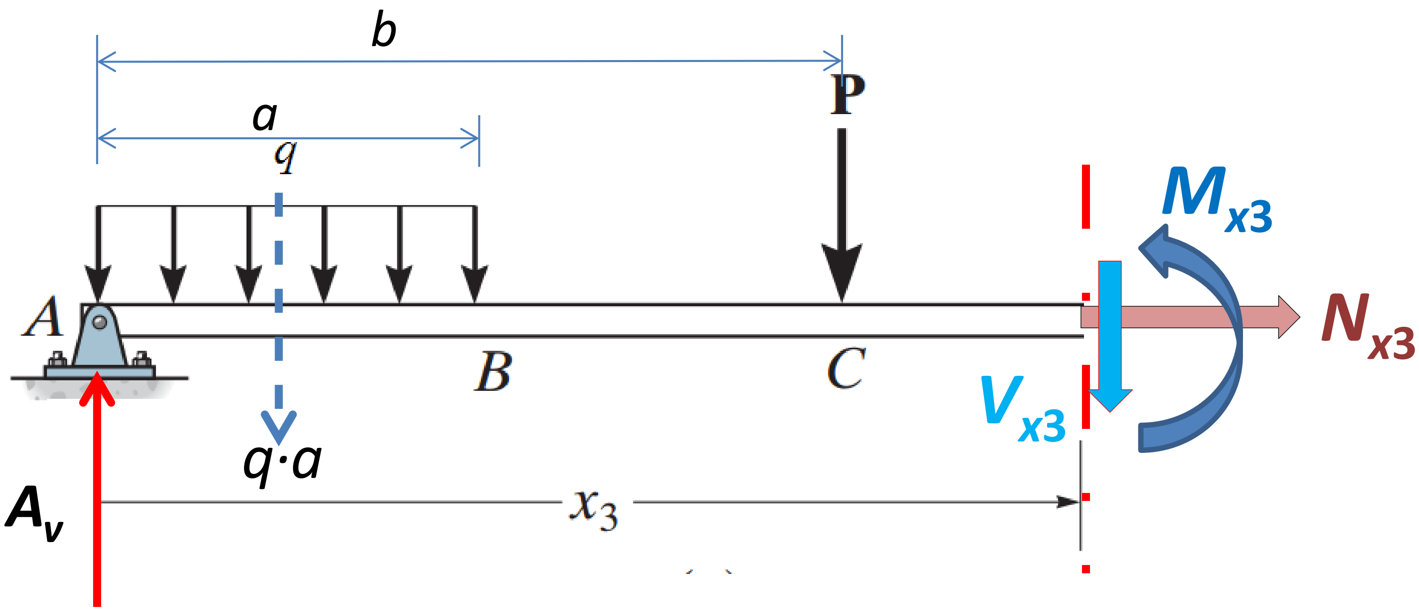
$+A_v \; ? \; q \cdot a - V_{x2} = 0$

$V_{x2} = A_v \; ? \; q \cdot a$

$\Sigma M_{x2} = 0$

$+A_v \cdot x_2 - q \cdot a(x_2 - a/2) - M_{x2} = 0$

$M_{x2} = A_v \cdot x_2 - q \cdot a(x_2 - a/2)$



For region C-D :

$\Sigma F_y = 0$

$+A_v$? $q \cdot a - P - V_{x3} = 0$

$V_{x3} = A_v$? $q \cdot a - P$

$\Sigma M_{x3} = 0$

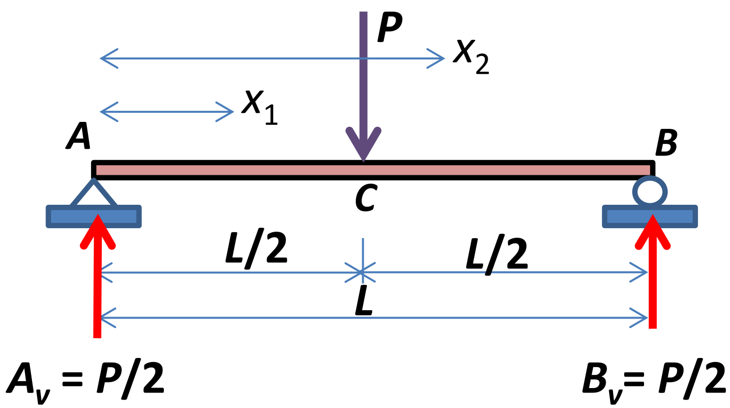
$+A_v \cdot x_3 - q \cdot a(x_3 - a/2) - P(x_3\text{-b}) - M_{x3} = 0$

$M_{x3} = A_v \cdot x_3 - q \cdot a(x_3 - a/2) - P(x_3\text{-b})$

- If the variations of *V* and *M* as functions of *x* are plotted, the graphs are termed the ***shear force diagram (SFD)*** and ***bending moment diagram (BMD),*** respectively.

Example 2

- Derive the shear and moment function for the beams shown in the figure, then draw the SFD and BMD



<u>For region A-C :</u>

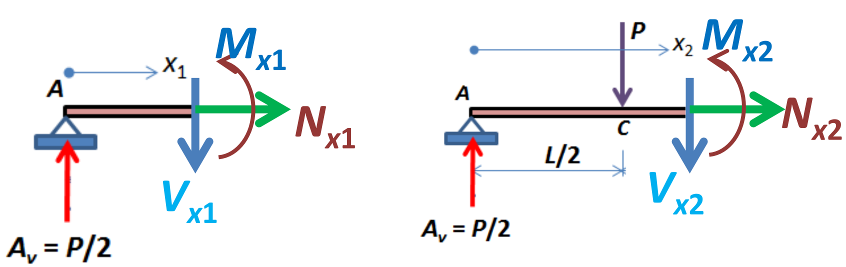
$\Sigma F_y = 0 \rightarrow \quad V_{x1} = A_v$

$\Sigma M_{x1} = 0 \rightarrow M_{x1} = A_v \cdot x_1$

<u>For region C-B :</u>

$\Sigma F_y = 0 \rightarrow \quad V_{x2} = A_v - P$

$\Sigma M_{x2} = 0 \rightarrow M_{x2} = A_v \cdot x_2 - P(x_2 - L/2)$



For region A-C :

$V_{x1} = A_v$

For $x_1 = 0$ $\quad V_{x1} = +A_v = +P/2$

For $x_1 = L/2$ $\quad V_{x1} = +A_v = +P/2$

$M_{x1} = A_v \cdot x_1$

For $x_1 = 0$ $\quad M_{x1} = 0$

For $x_1 = L/2$ $\quad M_{x1} = P/2(L/2) = +PL/4$

For region C-B :

$V_{x2} = A_v - P$

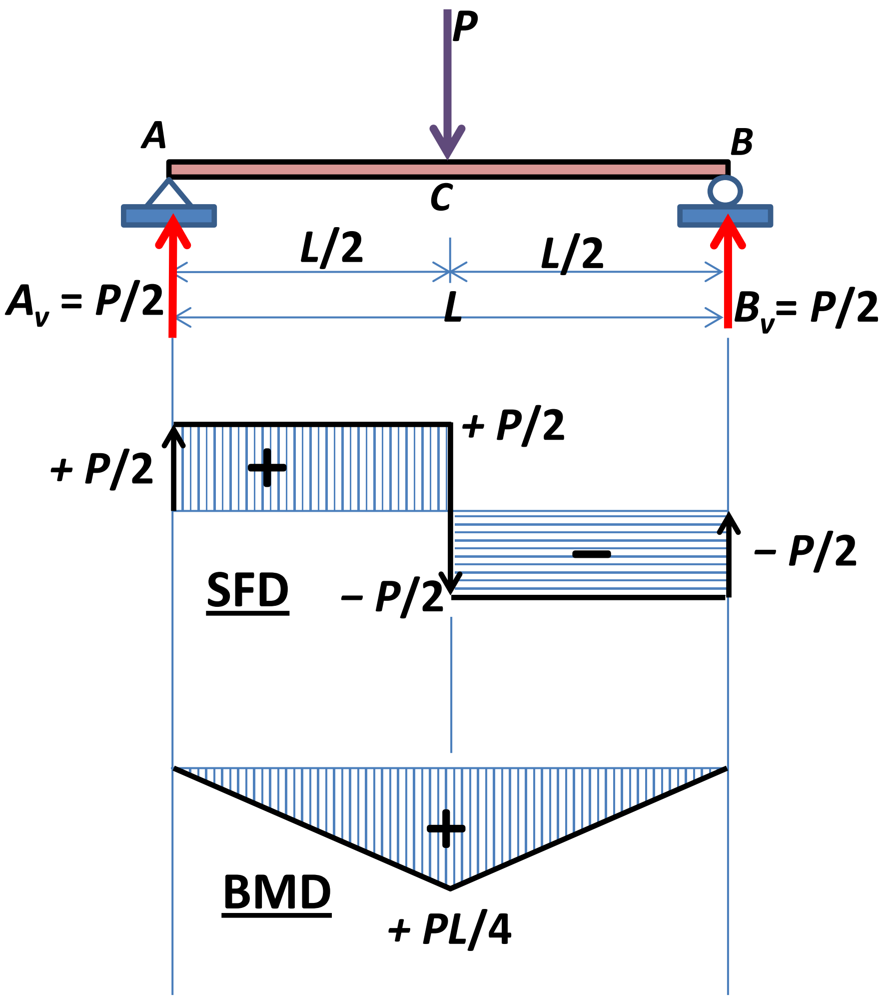
For $x_2 = L/2$ $\quad V_{x2} = -P/2$

For $x_2 = L$ $\quad V_{x2} = -P/2$

$M_{x2} = A_v \cdot x_2 - P(x_2 - L/2)$

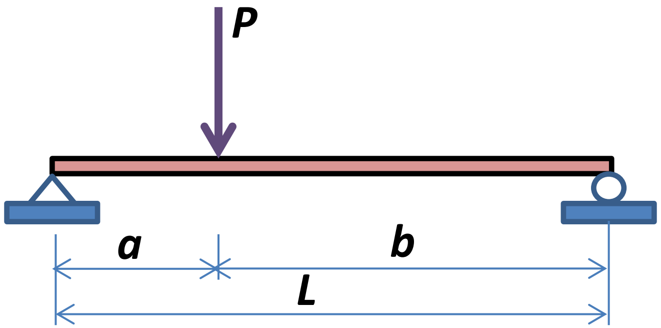
For $x_2 = L/2$ $\quad M_{x2} = +PL/4$

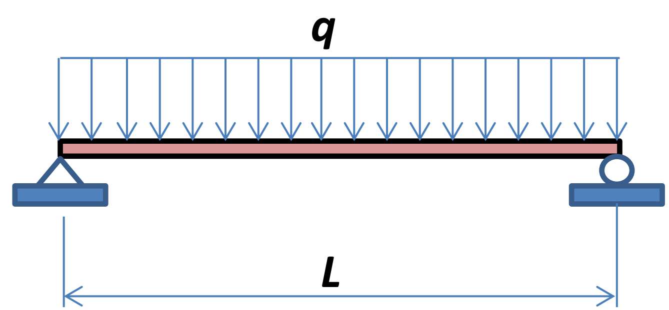
For $x_2 = L$ $\quad M_{x2} = 0$

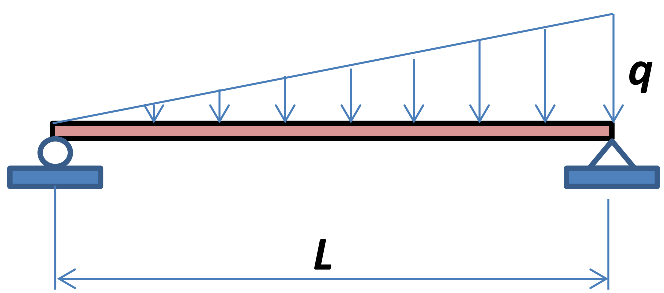


Example 3

- Derive the shear and moment function for the beams shown in the figure, then draw the SFD and BMD for each beam

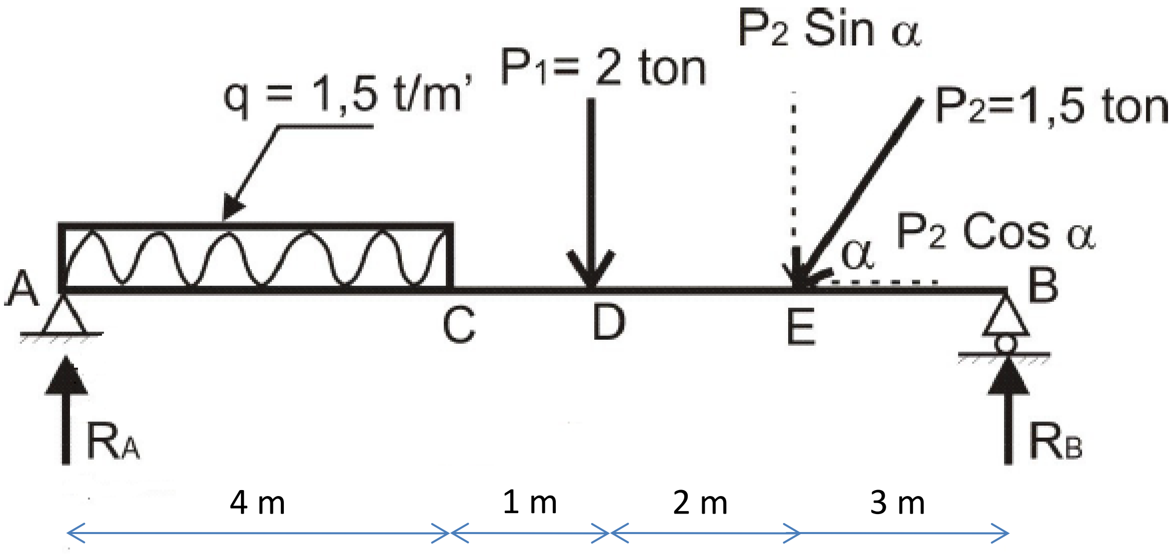






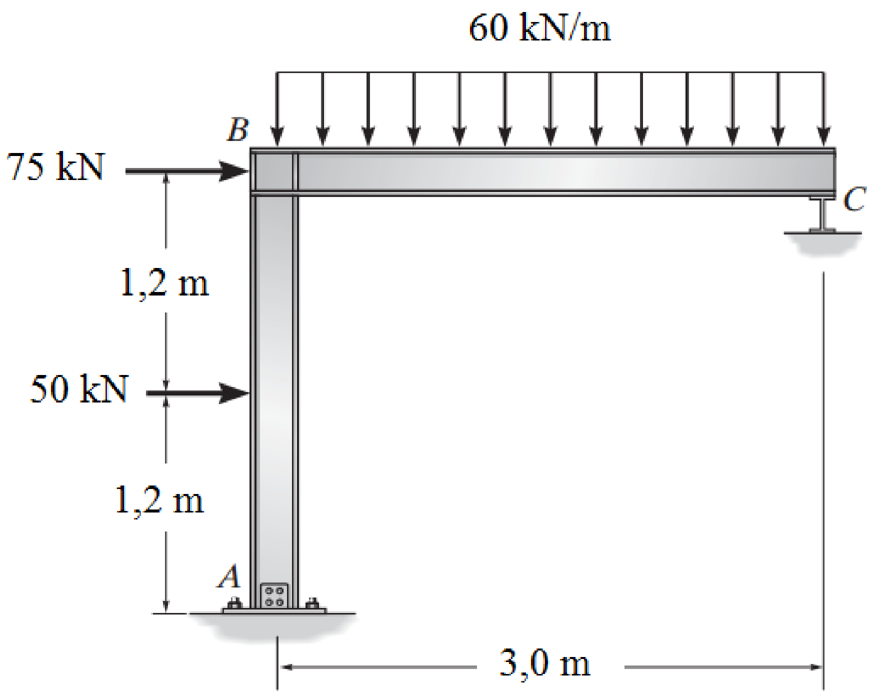
Example 4

- Draw the shear, normal and moment diagram for the beam in figure. (tan α = ¾)



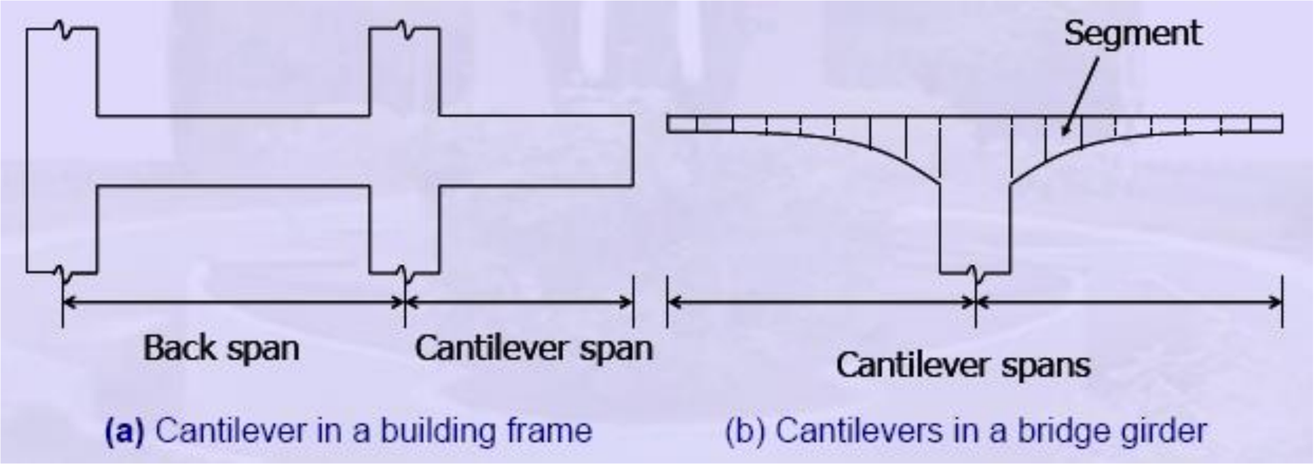
Example 5

- Draw the shear and moment diagrams for each member of the frame. Assume the frame is pin connected at *A*, and *C*



Balok Cantilever (overstek)

- Balok kantilever adalah sebuah balok yang memiliki perletakan (*support*) hanya pada salah satu ujungnya.



- Biasa ditemukan pada konstruksi bangunan gedung (balkon) dan juga konstruksi jembatan

Universitas Pembangunan Jaya

Integrity, Professionalism, & Entrepreneurship

Balok Kantilever dengan beban terpusat

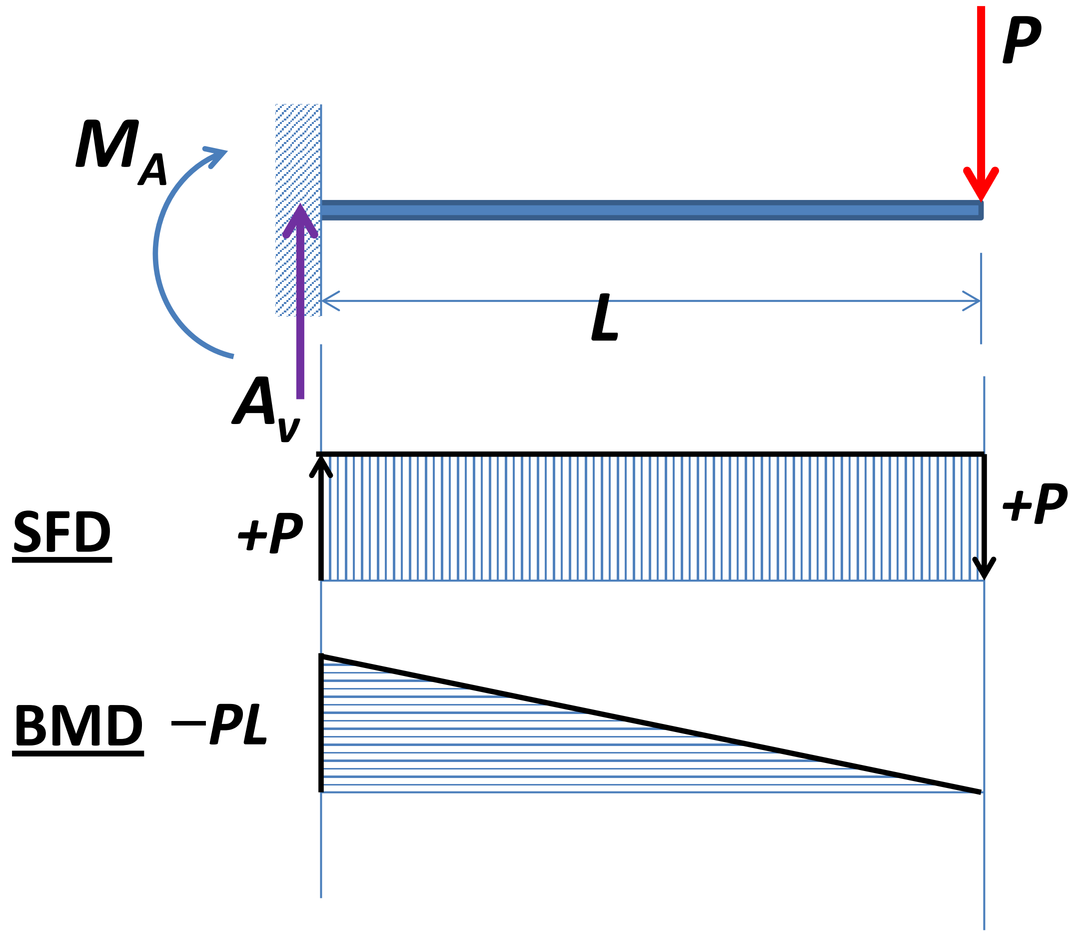
Reaksi Tumpuan :

$\sum F_y = 0$

$A_V = P$

$\sum M_A = 0$

$M_A = -PL$



Balok Kantilever dengan beban terbagi rata

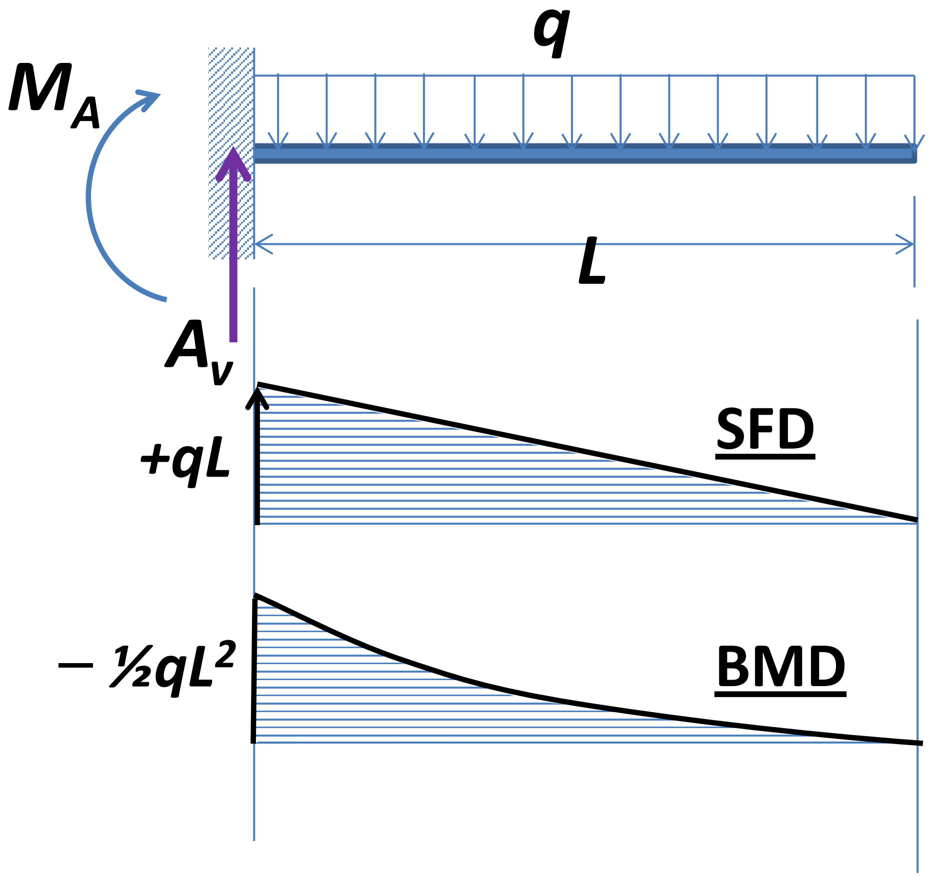
Reaksi Tumpuan :

$$\boldsymbol{\sum F_y = 0}$$

$$A_V = qL$$

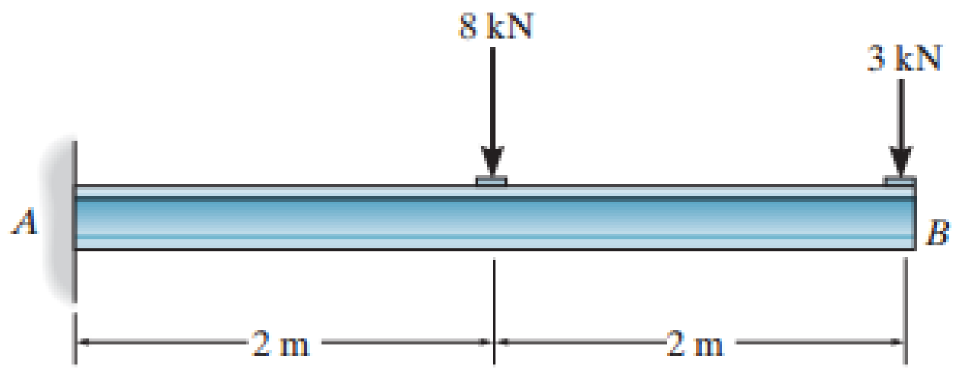
$$\boldsymbol{\sum M_A = 0}$$

$$M_A = -\tfrac{1}{2}qL^2$$



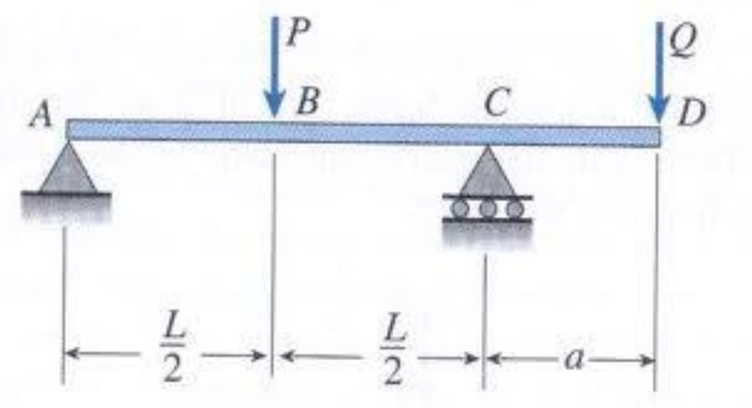
Example 1

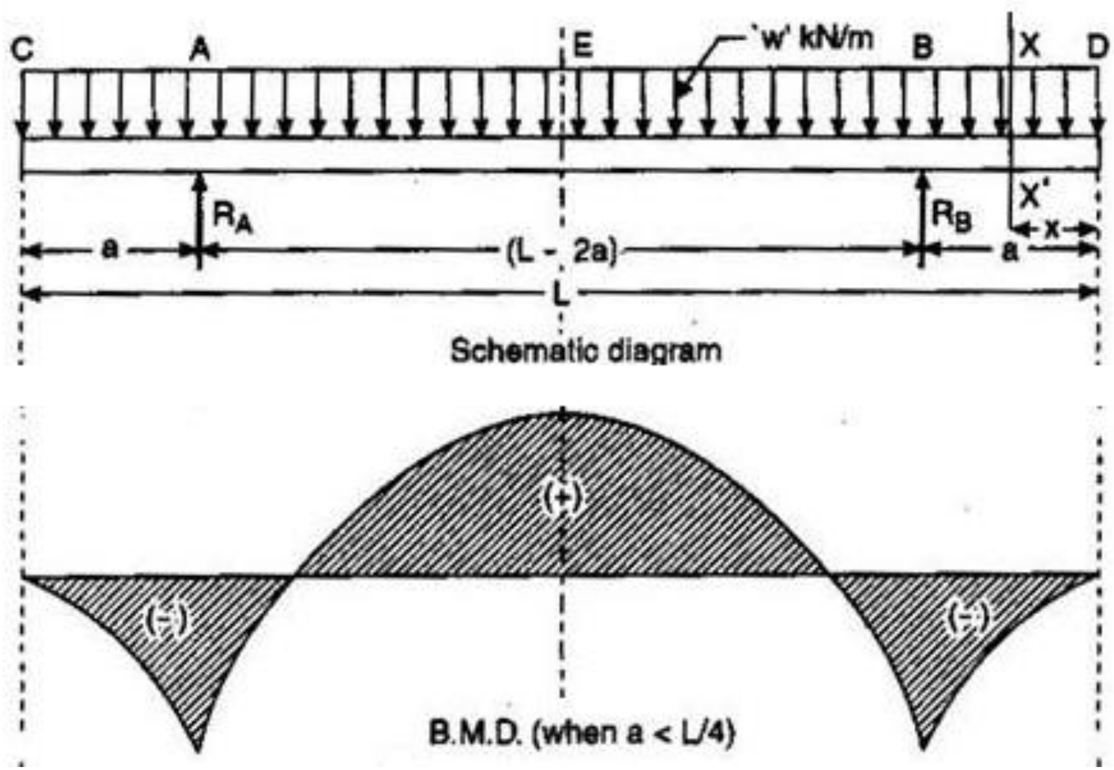
- Draw the shear and moment diagrams for the beam



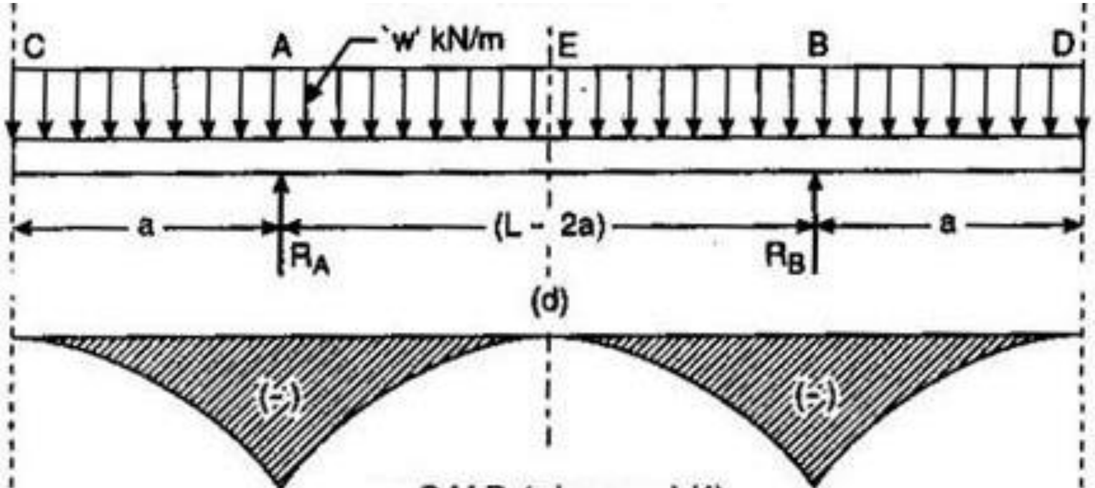
Simply Supported Beam with Overhanging

- Adalah balok yang salah satu atau kedua ujungnya bebas dan merupakan terusan dari tumpuan/perletakan. Kadang disebut juga kombinasi balok dan kantilever.
- Dapat menghasilkan momen negatif pada tumpuan tetapi struktur secara keseluruhan tetap struktur statis tertentu.

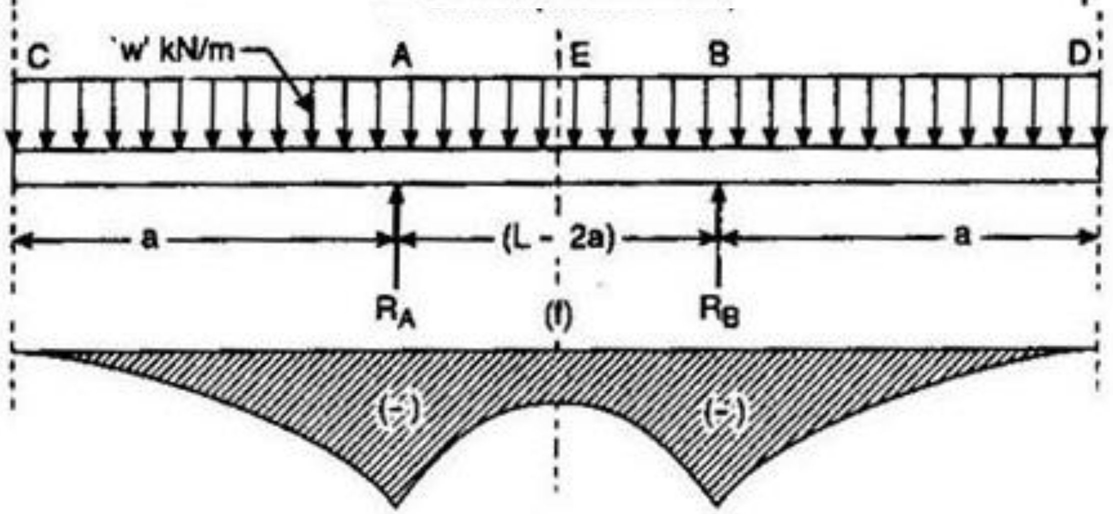




Distribusi Momen Pada Balok sederhana dengan *Overhanging*



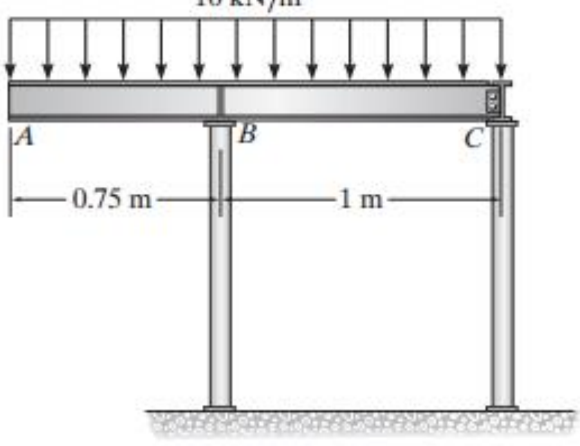
B.M.D. (when a = L/4)



B.M.D. (when a > L/4)

Example 2

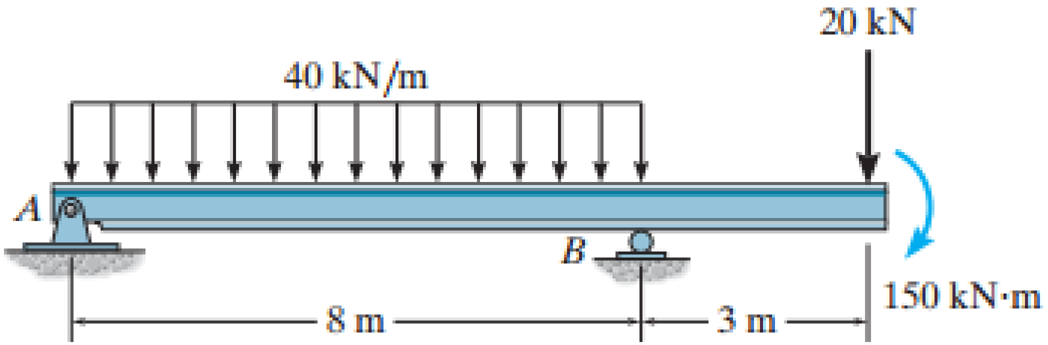
- The beam shown in the photo is used to support a portion of the overhang for the entranceway of the building. The idealized model for the beam with the load acting on it is shown in Fig. *a*. Assume *B* is a roller and *C* is pinned. Draw the shear and moment diagrams for the beam.



(a)

Example 3

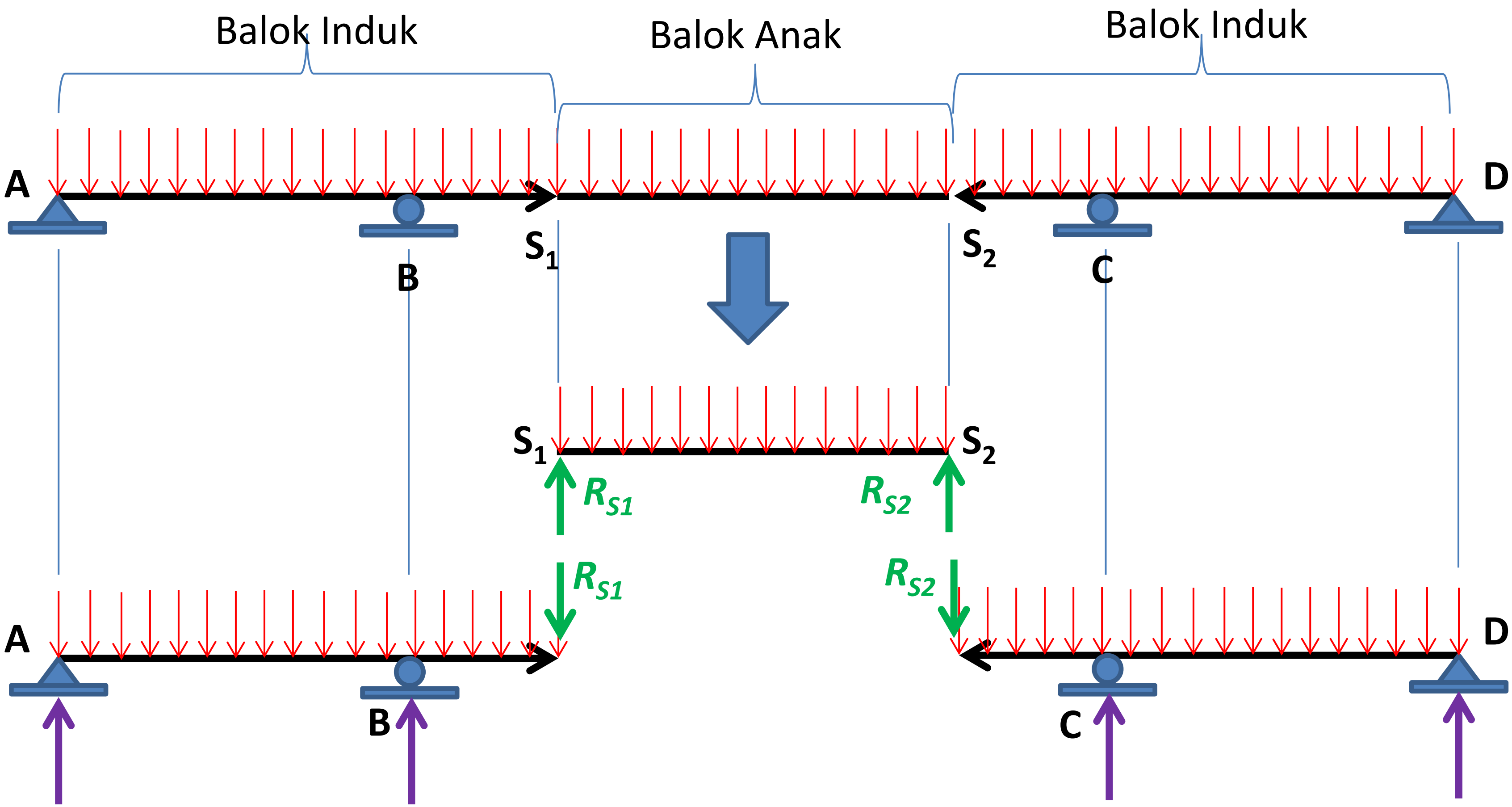
- Draw the shear and moment diagrams for the beam



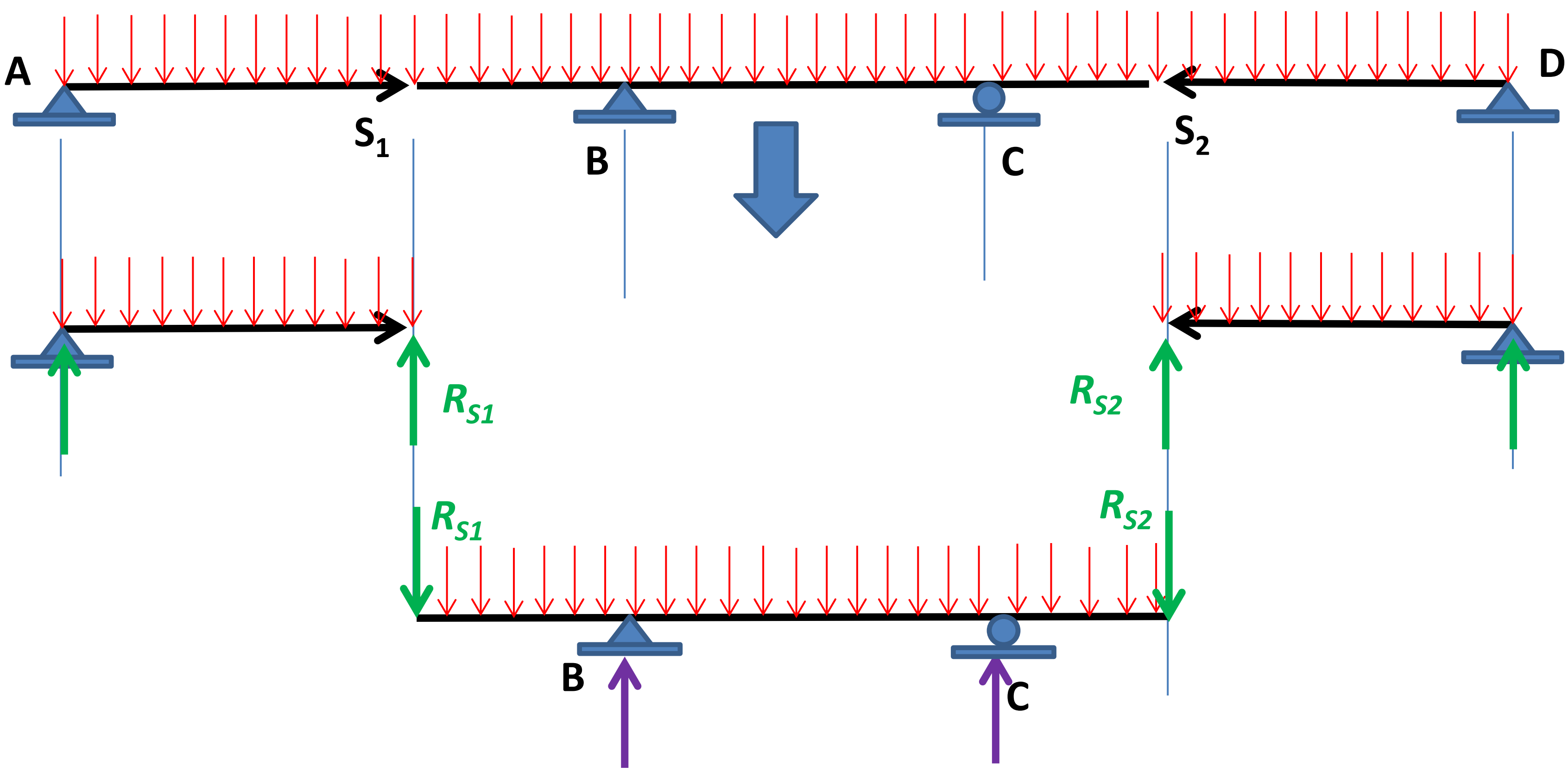
- Pada suatu struktur balok bentang panjang yang memiliki beberapa tumpuan, terkadang dapat dibuat kombinasi dari beberapa balok yang lebih pendek yang disebut balok gerber

- Balok gerber termasuk ke dalam struktur statis tertentu
- Balok gerber terdiri dari balok anak dan balok induk yang dihubungkan dengan sendi dalam (internal hinge)
- Langkah analisis balok gerber dimulai dengan menyelesaikan dahulu struktur balok anak
- Reaksi-reaksi tumpuan dari balok anak selanjutnya dilimpahkan pada balok induk.
- Reaksi tumpuan pada balok induk dapat dicari dengan menggunakan persamaan kesetimbangan biasa
- Selanjutnya diagram gaya lintang dan diagram momen dapat digambarkan

Konstruksi Balok Gerber (1)

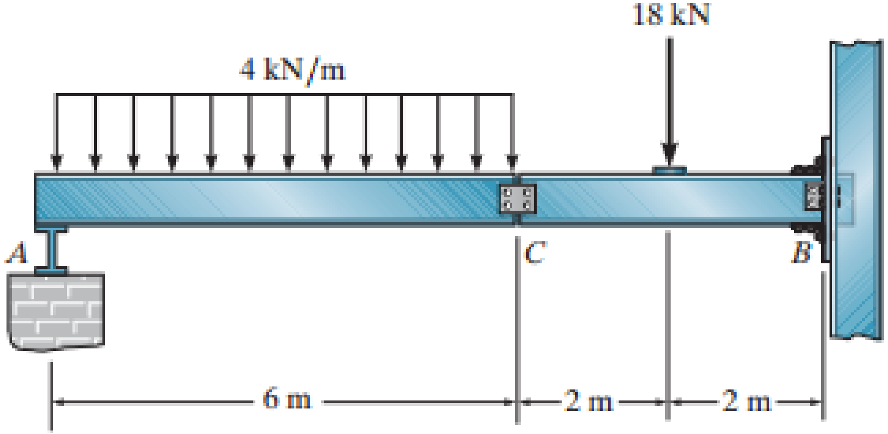


Konstruksi Balok Gerber (2)



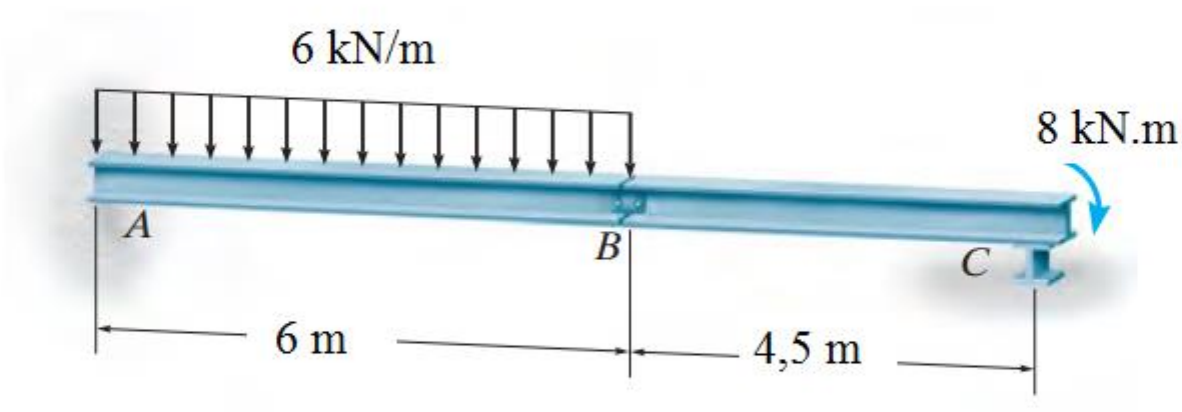
Example 1

- Determine the reactions at the supports *A* and *B* of the compound beam. Assume there is a pin at *C*. Draw the SFD and BMD



Example 2

- Determine the reactions at *A*, *B*, and *C*. Assume that the connection at *B* is a pin and *C* is a roller. Draw the SFD and BMD



Example 3

- Determine the reactions at *A* and C. The frame is pin connected at A, B, and C and the two joints are fixed connected.