Integral and derivative Table

In this table, a is a constant, while u, v, w are functions. The derivatives are expressed as derivatives with respect to an arbitrary variable x.

$$\frac{d}{dx}(a) = 0$$

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(au) = a\frac{du}{dx}$$

$$\frac{d}{dx}(u + v - w) = \frac{du}{dx} + \frac{dv}{dx} - \frac{dw}{dx}$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{1}{v}\frac{du}{dx} - \frac{u}{v^2}\frac{dv}{dx}$$

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$

$$\frac{d}{dx}(\sqrt{u}) = \frac{1}{2\sqrt{u}}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{1}{u}\right) = -\frac{1}{u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{1}{u^n}\right) = -\frac{n}{u^{n+1}}\frac{du}{dx}$$

$$\frac{d}{dx}[f(u)] = \frac{d}{du}[f(u)]\frac{du}{dx}$$

$$\frac{d}{dx}[\ln u] = \frac{d}{dx}[\log_e u] = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}[\log_a u] = \log_a e\frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}e^u = e^u\frac{du}{dx}$$

$$\frac{d}{dx}a^u = a^u \ln a\frac{du}{dx}$$

$$\frac{d}{dx}\left(u^v\right) = vu^{v-1}\frac{du}{dx} + \ln u \;\; u^v\frac{dv}{dx}$$

$$\frac{d}{dx}\sin u = \cos u\frac{du}{dx}$$

$$\frac{d}{dx}\cos u = -\sin u\frac{du}{dx}$$

$$\frac{d}{dx}\tan u = \sec^2 u\frac{du}{dx}$$

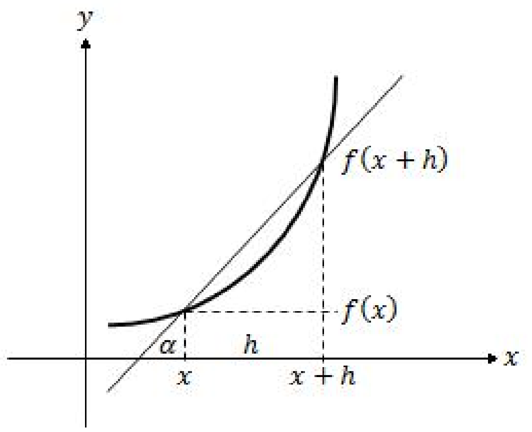
$$\frac{d}{dx}\cot u = -\csc^2 u\frac{du}{dx}$$

$$\frac{d}{dx}\sec u = \sec u\tan u\frac{du}{dx}$$

$$\frac{d}{dx}\csc u = -\csc u\cot u\frac{du}{dx}$$

http://www.ambrsoft.com/equations/Derivation.html

Derivation Basic Rules

If f is a function of the independent variable x, the derivative of the function is defined by the equation:	$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
	$f(x+h) - f(x)$ *is the height of the triangle.* h *is the base length of the triangle.* *The slope is:* $\tan\alpha = \frac{f(x+h) - f(x)}{(x+h) - x}$ *So when* h *tends to zero the expression become:* $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ *This is the slop of the tangent line to the function* $f(x)$ *at point* x.
chain rule: *Suppose that:* $y = y(u)$ *and* $u = u(x)$ *then* $\frac{dy}{dx}$ *is defined by:*	$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ $\quad du \neq 0$ *and* $dx \neq 0$
multiplication rule: *If* $f(x) = g(x) \cdot u(x)$ *then* f' *is:*	$f'(x) = g'u + gu'$
quotient rule: *If* $f(x) = \frac{g(x)}{u(x)}$ *then* f' *is:*	$f'(x) = \frac{g' \cdot u - g \cdot u'}{u^2}$ $\quad u \neq 0$
Reciprocal rule: *If* $f(x) = \frac{1}{u(x)}$ *then* f' *is:*	$f'(x) = -\frac{u'}{u^2}$ $\quad u \neq 0$
Addition rule: *If* $f = f(x)$ *and* $g = g(x)$ *and* a *and* b *are real numbers then* $(af + bg)'$ *is:*	$(af + bg)' = af' + bg'$
Constant rule: *If* $f(x)$ *is a constant then* f' *is:*	$f' = 0$
If $f = f(x)$ *then all the following notations for derivatives are valid:*	*First derivative:* $\frac{df}{dx} \equiv f' \equiv \dot{f} \equiv f_x$ *Second derivative:* $\frac{d^2 f}{dx^2} \equiv \frac{d}{dx}\left(\frac{df}{dx}\right) \equiv f'' \equiv \ddot{f} \equiv f_{xx}$

$\frac{d}{dx}(c) = 0$	$\frac{d}{dx}(cx) = c$	$\frac{d}{dx}(x^c) = cx^{c-1}$
$\frac{d}{dx}(c^x) = c^x \ln(c) \qquad c > 0$	$\frac{d}{dx}(x^x) = x^x(1 + ln\, x)$	$\frac{d}{dx}(e^x) = e^x$
$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$	$\frac{d}{dx}\left(\frac{1}{x^2}\right) = \frac{-2}{x^3}$	$\frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{-n}{x^{n+1}}$
$\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}} \qquad x > 0$	$\frac{d}{dx}(\sqrt[3]{x}) = \frac{1}{3 \cdot \sqrt[3]{x^2}}$	$\frac{d}{dx}(\sqrt[n]{x}) = \frac{1}{n \cdot \sqrt[n]{x^{n-1}}}$
$\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) = \frac{-1}{2\sqrt{x^3}}$	$\frac{d}{dx}\left(\frac{1}{\sqrt[3]{x}}\right) = \frac{-1}{3 \cdot \sqrt[3]{x^4}}$	$\frac{d}{dx}\left(\frac{1}{\sqrt[n]{x}}\right) = \frac{-1}{n \cdot \sqrt[n]{x^{n+1}}}$
$\frac{d}{dx}(\ln x) = \frac{1}{x} \qquad x > 0$	$\frac{d}{dx}(x \cdot \ln x) = \ln x + 1$	$\frac{d}{dx}(\log_c x) = \frac{1}{x \ln c} \qquad c > 0 \quad c \neq 1$
$\frac{d}{dx}\left(\frac{1}{ln x}\right) = \frac{-1}{x(\ln x)^2}$	$\frac{d}{dx}\left(\frac{1}{x \cdot \ln x}\right) = \frac{-(\ln x + 1)}{(x \cdot \ln x)^2}$	$\frac{d}{dx}\left(\frac{1}{\log_c x}\right) = \frac{-1}{x \cdot \ln c \cdot (\log_c x)^2}$
$\frac{d}{dx}\left(\frac{1}{x+1}\right) = \frac{-1}{(x+1)^2}$	$\frac{d}{dx}\left(\frac{1}{(x+1)^2}\right) = \frac{-2}{(x+1)^3}$	$\frac{d}{dx}\left(\frac{1}{(x+1)^n}\right) = \frac{-n}{(x+1)^{n+1}}$
$\frac{d}{dx}\left(\frac{1}{\sqrt{x+1}}\right) = \frac{-1}{2 \cdot \sqrt{(x+1)^3}}$	$\frac{d}{dx}\left(\frac{1}{\sqrt[3]{x+1}}\right) = \frac{-1}{3 \cdot \sqrt[3]{(x+1)^4}}$	$\frac{d}{dx}\left(\frac{1}{\sqrt[n]{x+1}}\right) = \frac{-1}{n \cdot \sqrt[n]{(x+1)^{n+1}}}$

[DERIVATION TABLES] --- Trigonometric Functions

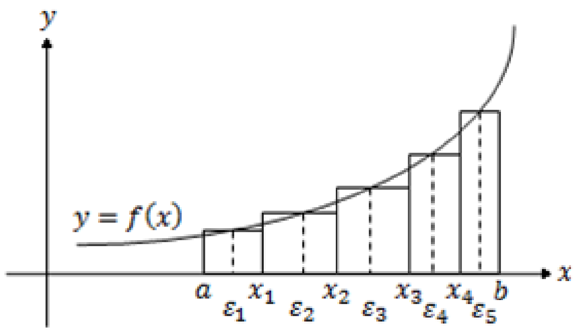
$\frac{d}{dx}\sin x = \cos x$	$\frac{d}{dx}\sinh x = \cosh x$
$\frac{d}{dx}\cos x = -\sin x$	$\frac{d}{dx}\cosh x = \sinh x$
$\frac{d}{dx}\tan x = \sec^2 x = \frac{1}{\cos^2 x}$	$\frac{d}{dx}\tanh x = 1 - \tanh^2 x = \text{sech}^2 x$
$\frac{d}{dx}\cot x = -\csc^2 x = -\frac{1}{\sin^2 x}$	$\frac{d}{dx}coth\, x = -csch^2 x$
$\frac{d}{dx}\csc x = -\csc x \cot x$	$\frac{d}{dx}\text{csch}\, x = -\text{csch}\, x \coth x$
$\frac{d}{dx}\sec x = sec\, x\, tan\, x$	$\frac{d}{dx}\text{sech}\, x = -\text{sech}\, x \tanh x$

$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$	$\lvert x\rvert < 1$	$\frac{d}{dx}\sinh^{-1} x = \frac{1}{\sqrt{1+x^2}}$	
$\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$	$\lvert x\rvert < 1$	$\frac{d}{dx}\cosh^{-1} x = \frac{1}{\sqrt{x^2-1}}$	
$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$		$\frac{d}{dx}\tanh^{-1} x = \frac{1}{1-x^2}$	$\lvert x\rvert < 1$
$\frac{d}{dx}\cot^{-1} x = \frac{-1}{1+x^2}$		$\frac{d}{dx}\coth^{-1} x = \frac{1}{1-x^2}$	$\lvert x\rvert < 1$
$\frac{d}{dx}\csc^{-1} x = \frac{-1}{x\sqrt{x^2-1}}$	$\lvert x\rvert > 1$	$\frac{d}{dx}\text{csch}^{-1} x = \frac{-1}{x\sqrt{x^2+1}}$	
$\frac{d}{dx}\sec^{-1} x = \frac{1}{x\sqrt{x^2-1}}$	$\lvert x\rvert > 1$	$\frac{d}{dx}\text{sech}^{-1} x = \frac{-1}{x\sqrt{x^2-1}}$	

Partial Derivation

Partial Derivatives definition: If f is a function of two variables or more $f = f(x, y)$, then the partial derivatives can be found according to both variables as: $\frac{\partial f}{\partial x}$ (y is kept constant) and $\frac{\partial f}{\partial y}$ (x kept constant)	$\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$ $\frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h}$
Chain rule: *for computing partial derivatives:* If $f = f(x,y)$ is continuous and both derivatives exists and $x = x(r, s)$ and $y = y(r, s)$ then:	$\frac{\partial f}{\partial r} = \left(\frac{\partial f}{\partial x}\right)\left(\frac{\partial x}{\partial r}\right) + \left(\frac{\partial f}{\partial y}\right)\left(\frac{\partial y}{\partial r}\right)$ $\frac{\partial f}{\partial s} = \left(\frac{\partial f}{\partial x}\right)\left(\frac{\partial x}{\partial s}\right) + \left(\frac{\partial f}{\partial y}\right)\left(\frac{\partial y}{\partial s}\right)$
If $f = f(x, y)$ and their derivatives f_x, f_y are continuous in the range of the derivation then:	$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} \quad (f_{xy} = f_{yx})$
If $f = f(x,y,z)$ and is differentiable in a range then the implicit differential is:	$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$
If $f = f(x,y,z)$ and the function is continuous in the domain, then the ∇ operator can be defined. The meaning of this operator is the gradient vector at the point (a).	$\nabla f(a) = \left(\frac{\partial f}{\partial x}(a) + \frac{\partial f}{\partial y}(a) + \frac{\partial f}{\partial z}(a)\right)$
Laplace operator in three dimentions: f should be twice differentiable in the domain.	$\Delta f = \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$
Laplace operator in polar form of two variables $f = f(r, \theta)$:	$\Delta f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2}$
Laplace operator in spherical form $f = f(r, \theta, \varphi)$ θ – azimuth, φ – polar angle:	$\Delta f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\varphi}\frac{\partial}{\partial \varphi}\left(\sin\varphi\frac{\partial f}{\partial \varphi}\right) + \frac{1}{r^2 \sin^2\varphi}\frac{\partial^2 f}{\partial \theta^2}$
Second and higher derivatives notation: If we write $z = f(x,y)$, then the following symbols have the same meanings:	$\frac{\partial^2 z}{\partial x^2}; \quad \frac{\partial^2 f}{\partial x^2}; \quad \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right); \quad f_{xx}; \quad z_{xx}$

Integrals Basic Data

	***Definite integral*** *The definite integral of a function from point a to point b is equivalent to the area under the graph and the x axis.* *The integral can be calculated by findding the sum of each rectangle area:* *First rectangle area is:* $f(\varepsilon_1)\cdot(x_1-a)$ *Second rectangle area is:* $f(\varepsilon_2)\cdot(x_2-x_1)$ *If* $\Delta x_k = x_k - x_{k-1}$ *then the area is:* $area = \lim_{\Delta x_k \to 0} \sum_{k=1}^{n} f(\varepsilon_k)\cdot \Delta x_k = \int_a^b f(x)dx$
If $f=f(x)$ *and* $g=g(x)$ *then:*	$\int_a^b (f(x) \pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$
If $f=f(x)$ *is defined in the range a to b and c is a point inside this range then:*	$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
If C is a constant then:	$\int_a^b Cf(x)dx = C\int_a^b f(x)dx$
Integration ranges can be changed according to the rule:	$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
To the value received after integration always add a term of a constant C (this term is omitted in the following integral tables).	$\int a dx = ax + C$
Integration between two equals points are zero.	$\int_a^a f(x)dx = 0$

Integration by parts: because $d(uv) = udv + vdu$ we can integrate both sides:	$\int u\,dv = uv - \int v\,du$
Example: find $\int xe^x dx$	Write this integral in the form $\int udv$ $u = x$ and $dv = e^x dx$ then $du = dx$ and $v = e^x$ The integration is: $\int xe^x\,dx = xe^x - \int e^x dx$ $= xe^x - e^x + C = e^x(x-1) + C$
Integration by substitution: If f is a continuous function, then:	$\int_a^b f(g(t))\,g'(t)dt = \int_{g(a)}^{g(b)} f(x)\,dx$
Example: find $\int_0^2 \frac{(2x+1)dx}{\sqrt{x^2+x+1}}$	Substitute: $u = x^2 + x + 1$ Then $du = (2x+1)dx$ $\int_1^7 \left(\frac{(2x+1)du}{\sqrt{u}(2x+1)}\right) = \int_1^7 \frac{du}{\sqrt{u}} = 2\sqrt{u}\,\Big\|_0^7 = 2(\sqrt{7}-1)$ The new integration limits are: $u(x=0) = 1$ and $u(x=2) = 7$ Note: we could substitute the value $u = u(x)$ in the integral result and then leave the old integration limits.

[INTEGRAL TABLES] --- Basic functions of the forms: x^n e^x xe^x $e^x \sin x$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} \qquad n \neq -1$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert$
$\int a^x\,dx = \frac{a^x}{\ln a} \qquad a > 0,\ a \neq 1$	$\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x}$
$\int e^x\,dx = e^x$	$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$
$\int x^2e^2\,dx = e^x(x^2 - 2x + 2)$	$\int x^2e^{ax}\,dx = e^{ax}\left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)$
$\int x^ne^x\,dx = x^ne^x - n\int x^{n-1}e^x\,dx$	$\int x^ne^{ax}\,dx = \frac{1}{a}x^ne^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx$
$\int xe^x\,dx = (x-1)e^x$	$\int xe^{ax}\,dx = \frac{1}{a}xe^{ax} - \frac{1}{a^2}e^{ax}$

$\int e^x \sin x\,dx = \frac{1}{2}e^x(\sin x - \cos x)$	$\int e^x \cos x\,dx = \frac{1}{2}e^x(\sin x + \cos x)$
$\int e^{ax}\sin(bx)\,dx = \frac{e^{ax}(a\sin bx - b\cos bx)}{a^2+b^2}$	$\int e^{ax}\cos(bx)\,dx = \frac{e^{ax}(b\sin bx + a\cos bx)}{a^2+b^2}$
$\int xe^x \sin x\,dx = \frac{1}{2}e^x(\cos x + x\sin x - x\cos x)$	$\int xe^x \cos x\,dx = \frac{1}{2}e^x(x\sin x + x\cos x - \sin x)$
$\int e^x \tanh x\,dx = e^x - 2\tan^{-1}(e^x)$	
$\int e^{ax}\sinh(bx)\,dx = \frac{e^{ax}(a\sinh(bx) - b\cosh(bx))}{a^2-b^2}$	$\int e^{ax}\cosh(bx)\,dx = \frac{e^{ax}(-b\sinh(bx) + a\cosh(bx))}{a^2-b^2}$

[INTEGRAL TABLES] --- Logarithms functions of the forms: $\ln x$

$\int \ln\lvert x\rvert\ \,dx = x\ln x - x$	$\int (\ln\lvert x\rvert)^2\,dx = x(\ln\lvert x\rvert)^2 - 2x\ln\lvert x\rvert + 2x$
$\int x^n\ln\lvert x\rvert\ \,dx = \frac{x^{n+1}}{n+1}\ln\lvert x\rvert - \frac{x^{n+1}}{(n+1)^2} \qquad n \neq -1$	$\int \frac{\ln^2 x}{x}\,dx = \frac{\ln^3 x}{3}$
$\int x^n\ln(ax)\,dx = \frac{1}{n+1}x^{n+1}\ln(ax) - \frac{1}{(n+1)^2}x^{n+1} \qquad n \neq -1$	
$\int \ln(ax)\,dx = x\ln(ax) - x$	$\int \frac{\ln(ax)}{x}\,dx = \frac{1}{2}[\ln(ax)]^2$
$\int \ln(ax+b)\,dx = \frac{ax+b}{a}\ln(ax+b) - x$	$\int x\ln(ax+b)\,dx = \frac{b}{2a}x - \frac{1}{4}x^2 + \frac{1}{2}\left(x^2 - \frac{b^2}{a^2}\right)\ln(ax+b)$
$\int \log_a x\,dx = \int \frac{\ln x}{\ln a}\,dx = \frac{x\ln x - x}{\ln a}$	$\int \frac{1}{x\ln x}\,dx = \ln(\ln x)$

[INTEGRAL TABLES] --- Trigonometric functions of the forms: $\sin x$

$\int \sin x\,dx = -\cos x$	$\int \sin^2 x\,dx = \frac{1}{2}x - \frac{1}{4}\sin(2x)$	$\int \sin^{-1} x\,dx = x\sin^{-1} x + \sqrt{1-x^2}$
$\int \cos x\,dx = \sin x$	$\int \cos^2 x\,dx = \frac{x}{2} + \frac{1}{4}\sin(2x)$	$\int \cos^{-1} x\,dx = x\cos^{-1} x - \sqrt{1-x^2}$
$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \tan^2 x\,dx = \tan x - x$	$\int \tan^{-1} x\,dx = x\tan^{-1} x - \ln\left(\sqrt{1+x^2}\right)$
$\int \csc x\,dx = \ln\left\lvert\tan\frac{x}{2}\right\rvert$	$\int \csc^2 x\,dx = -\cot x$	$\int \csc^{-1} x\,dx = x\csc^{-1} x + \ln\left(x+\sqrt{x^2-1}\right)$
$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert$	$\int \sec^2 x\,dx = \tan x$	$\int \sec^{-1} x\,dx = x\sec^{-1} x - \ln\left(x+\sqrt{x^2-1}\right)$
$\int \cot x\,dx = \ln\lvert\sin x\rvert$	$\int \cot^2 x\,dx = -\cot x - x$	$\int \cot^{-1} x\,dx = x\cot^{-1} x + \ln\left(\sqrt{1+x^2}\right)$
$\int \sinh x\,dx = \cosh x$	$\int \sinh^2 x\,dx = \frac{\sinh(2x)}{4}$	$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$
$\int \cosh x\,dx = \sinh x$	$\int \cosh^2 x\,dx = \frac{x}{2} + \frac{\sinh(2x)}{4}$	$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$
$\int \tanh x\,dx = \ln\lvert\cosh x\rvert$	$\int \tanh^2 x\,dx = x - \tanh x$	$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax)$
$\int \operatorname{csch} x\,dx = -\coth^{-1}(\cosh x)$	$\int \operatorname{csch}^2 x\,dx = -\coth x$	$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b)$
$\int \operatorname{sech} x\,dx = \tan^{-1}(\sinh x)$	$\int \operatorname{sech}^2 x\,dx = \tanh x$	$\int \sin^{-1}(ax)\,dx = x\sin^{-1}(ax) + \sqrt{a^2-x^2}$
$\int \coth x\,dx = \ln\lvert\sinh x\rvert$	$\int \tanh(ax)\,dx = \frac{1}{a}\ln(\cosh ax)$	$\int \tan^{-1}(ax)\,dx = x\tan^{-1}(ax) + \frac{1}{2a}\ln(1+a^2x^2)$

$\int \sin x \cos x \, dx = \frac{1}{2}\sin^2 x$	$\int \sec x \csc x \, dx = \ln(\tan x)$
$\int \sec x \tan x \, dx = \sec x$	$\int \csc x \cot x \, dx = -\csc x$
$\int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x$	$\int \operatorname{csch} x \coth x \, dx = -\operatorname{csch} x$
$\int \sin mx \cdot \sin nx \; dx = \frac{\sin(m-n)\,x}{2(m-n)} - \frac{\sin(m+n)\,x}{2(m+n)},$	$m \neq \pm n$
$\int \cos mx \cdot \cos nx \; dx = \frac{\sin(m-n)\,x}{2(m-n)x} + \frac{\sin(m+n)\,x}{2(m+n)},$	$m \neq \pm n$
$\int \sin mx \cdot \cos nx \; dx = -\frac{\cos(m-n)\,x}{2(m-n)} - \frac{\cos(m+n)\,x}{2(m+n)},$	$m \neq \pm n$
$\int \sinh(ax)\cosh(ax)\,dx = \frac{-2ax + \sinh(2ax)}{4a}$	
$\int \sin(ax)\sinh(bx)\,dx = \frac{b\cosh(bx)\sin(ax) - a\cos(ax)\sinh(bx)}{a^2+b^2}$	
$\int \sin(ax)\cosh(bx)\,dx = \frac{-a\cos(ax)\cosh(bx) + b\sin(ax)\sinh(bx)}{a^2+b^2}$	
$\int \sinh(ax)\cosh(bx)\,dx = \frac{b\cosh(bx)\sinh(ax) - a\cosh(ax)\sinh(bx)}{b^2-a^2}$	
$\int \cos(ax)\cosh(bx)\,dx = \frac{a\sin(ax)\cosh(bx) + b\cos(ax)\sinh(bx)}{a^2+b^2}$	
$\int \cos(ax)\sinh(bx)\,dx = \frac{b\cos(ax)\cosh(bx) + a\sin(ax)\sinh(bx)}{a^2+b^2}$	

$\int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx$	$\int \cos^n x \, dx = \frac{\sin x \cos^{n-1} x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx$
$\int \frac{dx}{\sin^n x} = -\frac{\cos x}{(n-1)\sin^{n-1} x} + \frac{n-2}{n-1} \int \frac{dx}{\sin^{n-2} x}, \quad n \neq 1$	$\int \frac{dx}{\cos^n x} = \frac{\sin x}{(n-1)\cos^{n-1} x} + \frac{n+2}{n-1} \int \frac{dx}{\cos^{n-2} x}$
$\int \sin^3 x \, dx = -\frac{3}{4}\cos x + \frac{1}{12}\cos(3x)$	$\int \cos^3 x \, dx = \sin x - \frac{\sin^3 x}{3}$
$\int \tan^3 x \, dx = \ln(\cos x) + \frac{1}{2}\sec^2 x$	$\int \csc^3 x \, dx = -\frac{1}{2}\cot x \csc x + \frac{1}{2}\ln\lvert \csc x - \cot x \rvert$

$\int x \sin x \, dx = \sin x - x \cos x$	$\int x \cos x \, dx = \cos x + x \sin x$
$\int x \sin(ax) \, dx = \frac{1}{a^2}\sin(ax) - \frac{x}{a}\cos(ax)$	$\int x \cos(ax) \, dx = \frac{1}{a} x \sin(ax) + \frac{1}{a^2}\cos(ax)$
$\int x^2 \sin x \, dx = 2x \sin x + (2 - x^2)\cos x$	$\int x^2 \cos x \, dx = 2x \cos x + (x^2 - 2)\sin x$
$\int x^2 \sin(ax) \, dx = \frac{2}{a^2} x \sin(ax) + \frac{2 - a^2 x^2}{a^3}\cos(ax)$	$\int x^2 \cos(ax) \, dx = \frac{2}{a^2} x \cos(ax) + \frac{a^2 x^2 - 2}{a^3}\sin(ax)$
$\int x^n \sin x \, dx = -x^n \cos x + n \int x^{n-1} \cos x \, dx$	$\int x^n \cos x \, dx = x^n \sin x - n \int x^{n-1} \sin x \, dx$
$\int x^n \sin(ax) \, dx = -\frac{1}{a} x^n \cos(ax) + \frac{n}{a} \int x^{n-1} \cos(ax) \, dx$	$\int x^n \cos(ax) \, dx = \frac{1}{a} x^n \sin(ax) - \frac{n}{a} \int x^{n-1} \sin(ax) \, dx$
$\int x \sin^{-1} x \, dx = \frac{1}{4}\left[(2x^2 - 1)\sin^{-1} x + x\sqrt{1 - x^2}\right]$	$\int x \cos^{-1} x \, dx = \frac{1}{4}\left[(2x^2 - 1)\cos^{-1} x - x\sqrt{1 - x^2}\right]$

[INTEGRAL TABLES] --- Algebric functions of the forms: $1 \pm x^2$

$\int \frac{1}{1 + x^2} dx = \tan^{-1} x$	$\int \frac{1}{\sqrt{1 + x^2}} dx = \sinh^{-1} x$
$\int \frac{1}{1 - x^2} dx = \tanh^{-1} x$	$\int \frac{1}{\sqrt{1 - x^2}} dx = \sin^{-1} x$
$\int \frac{-1}{\sqrt{1 - x^2}} dx = \cos^{-1} x$	$\int \frac{1}{\sqrt{x^2 - 1}} dx = \cosh^{-1} x$

[INTEGRAL TABLES] --- Algebric functions of the forms: $x + a, \quad x - a, \quad a + bx$

$\int (x+a)^n dx = (x+a)^n\left(\frac{a}{1+n}+\frac{x}{1+n}\right), \quad n \neq -1$	$\int \frac{1}{(x+a)^2} dx = \frac{-1}{x+a}$
$\int x(x+a)\, dx = \frac{(x+a)^{1+n}(nx+x-a)}{(n+2)(n+1)}$	$\int \frac{1}{x(x+a)} dx = \frac{1}{a}\ln\left(\frac{x}{x+a}\right)$
$\int \frac{x}{\sqrt{x+a}} dx = \frac{2}{3}(x+2a)\sqrt{x+a}$	$\int \sqrt{\frac{x}{x+a}} dx = \sqrt{x}\sqrt{x+a} - a\ln\left(\sqrt{x}+\sqrt{x+a}\right)$

$\int \sqrt{x-a}\, dx = \frac{2}{3}(x-a)^{\frac{3}{2}}$	$\int \frac{1}{\sqrt{x-a}} dx = 2\sqrt{x-a}$
$\int x\sqrt{x-a}\, dx = \frac{2}{3}a(x-a)^{\frac{3}{2}} + \frac{2}{5}(x-a)^{\frac{5}{2}}$	$\int \frac{x}{\sqrt{x-a}} dx = \frac{2}{3}(x-2a)\sqrt{x-a}$

$\int \frac{1}{\sqrt{a-x}} dx = 2\sqrt{a-x}$	$\int \sqrt{\frac{x}{a-x}} dx = -\sqrt{x}\sqrt{a-x} - a\tan^{-1}\left(\frac{\sqrt{x}\sqrt{a-x}}{x-a}\right)$

$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{b(n+1)} \qquad n \neq -1$	$\int \frac{1}{ax+b} dx = \frac{1}{a}\ln\lvert ax+b\rvert$
$\int \frac{x\, dx}{ax+b} = \frac{x}{a} - \frac{b}{a^2}\ln(ax+b)$	$\int \frac{dx}{x(ax+b)} = \frac{1}{b}\ln\left\lvert\frac{x}{ax+b}\right\rvert$
$\int \frac{x\, dx}{(ax+b)^2} = \frac{b}{a^2}\left(\frac{1}{ax+b} + \frac{1}{b}\ln\lvert ax+b\rvert\right)$	$\int \frac{dx}{x(ax+b)^2} = \frac{1}{b(ax+b)} + \frac{1}{b^2}\ln\left\lvert\frac{x}{ax+b}\right\rvert$
$\int \frac{1}{x^2(ax+b)} dx = -\frac{1}{bx} + \frac{a}{b^2}\ln\left(\frac{ax+b}{x}\right)$	$\int \frac{dx}{x^2(ax+b)^2} = \frac{-a}{b^2(ax+b)} - \frac{1}{b^2x} + \frac{2a}{b^3}\ln\left(\frac{ax+b}{x}\right)$
$\int \sqrt{ax+b}\, dx = \left(\frac{2b}{3a} + \frac{2x}{3}\right)\sqrt{ax+b}$	$\int \frac{dx}{\sqrt{ax+b}} = \frac{2\sqrt{ax+b}}{a}$
$\int x\sqrt{ax+b}\, dx = \frac{2(3ax-2b)\sqrt{(ax+b)^3}}{15a^2}$	$\int \frac{x\, dx}{\sqrt{ax+b}} = \frac{2(ax-2b)}{3a^2}\sqrt{ax+b}$
$\int \frac{\sqrt{ax+b}}{x} dx = 2\sqrt{ax+b} + b\int \frac{dx}{x\sqrt{ax+b}}$	$\int \frac{dx}{x\sqrt{ax+b}} = \frac{1}{\sqrt{b}}\ln\left\lvert\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right\rvert$

$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) \qquad a \neq 0$	$\int \frac{x}{a^2+x^2}\,dx = \frac{1}{2}\ln(a^2+x^2)$
$\int \frac{x^2}{a^2+x^2}\,dx = x - a\tan^{-1}\left(\frac{x}{a}\right)$	$\int \frac{x^3}{a^2+x^2}\,dx = \frac{1}{2}x^2 - \frac{1}{2}a^2\ln(a^2+x^2)$
$\int \frac{1}{x(a^2+x^2)}\,dx = \frac{1}{2a^2}\ln\left(\frac{x^2}{a^2+x^2}\right)$	$\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\ln\left\lvert x+\sqrt{a^2+x^2}\right\rvert$
$\int \frac{\sqrt{a^2+x^2}}{x}\,dx = \sqrt{a^2+x^2} - a\ln\left\lvert\frac{a+\sqrt{a^2+x^2}}{x}\right\rvert$	$\int x\sqrt{a^2+x^2}\,dx = \frac{1}{3}(a^2+x^2)^{\frac{3}{2}}$
$\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \ln\left(x+\sqrt{a^2+x^2}\right) = \sinh^{-1}\left(\frac{x}{a}\right)$	$\int \frac{1}{\lvert x\rvert\sqrt{a^2+x^2}}\,dx = -\frac{1}{a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right), \qquad x \neq 0$
$\int \frac{x}{\sqrt{a^2+x^2}}\,dx = \sqrt{a^2+x^2}$	$\int \frac{x^2}{\sqrt{a^2+x^2}}\,dx = \frac{1}{2}x\sqrt{a^2+x^2} - \frac{1}{2}\ln\left(x+\sqrt{a^2+x^2}\right)$

$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} \qquad \lvert x\rvert < a$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right) = \frac{1}{a}\tanh^{-1}\frac{x}{a} \qquad x^2 < a^2$
$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} \qquad a > \lvert x\rvert$	$\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right) \qquad 0 < x < a$
$\int \frac{\sqrt{a^2-x^2}}{x}\,dx = \sqrt{a^2-x^2} - a\ln\left\lvert\frac{a+\sqrt{a^2-x^2}}{x}\right\rvert$	$\int \frac{x}{\sqrt{a^2-x^2}}\,dx = -\sqrt{a^2-x^2}$
	$\int \frac{x^2}{\sqrt{a^2-x^2}}\,dx = -\frac{1}{2}x\sqrt{a-x^2} - \frac{1}{2}a^2\tan^{-1}\left(\frac{x\sqrt{a^2-x^2}}{x^2-a^2}\right)$

$\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left\lvert x+\sqrt{x^2-a^2}\right\rvert$	$\int \frac{1}{x^2-a^2}\,dx = -\frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)$
$\int \frac{\sqrt{x^2-a^2}}{x}\,dx = \sqrt{x^2-a^2} - a\cos^{-1}\left(\frac{a}{x}\right), \quad 0 < a < \lvert x\rvert$	$\int x\sqrt{x^2-a^2}\,dx = \frac{1}{3}(x^2-a^2)^{\frac{3}{2}}$
$\int \frac{x}{\sqrt{x^2-a^2}}\,dx = \sqrt{x^2-a^2}$	$\int \frac{x^2}{\sqrt{x^2-a^2}}\,dx = \frac{1}{2}x\sqrt{x^2-a^2} + \frac{1}{2}\ln\left(x+\sqrt{x^2-a^2}\right)$
$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right), \qquad \lvert x\rvert > a$	$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left\lvert x+\sqrt{x^2-a^2}\right\rvert \qquad \lvert x\rvert > a > 0$