

Calculus 1

Lecture 12:

Integrals: Calculating Area

By:

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Integral

$$F(x) = \int f(x) dx$$

Example: $f(x) = 2$

Indefinite integral: $\int 2 dx = 2x + c$

Definite integral: $\int_1^5 2 dx = 2x \Big|_1^5 = 2(5 - 1) = 8$

Definite Integral (1)

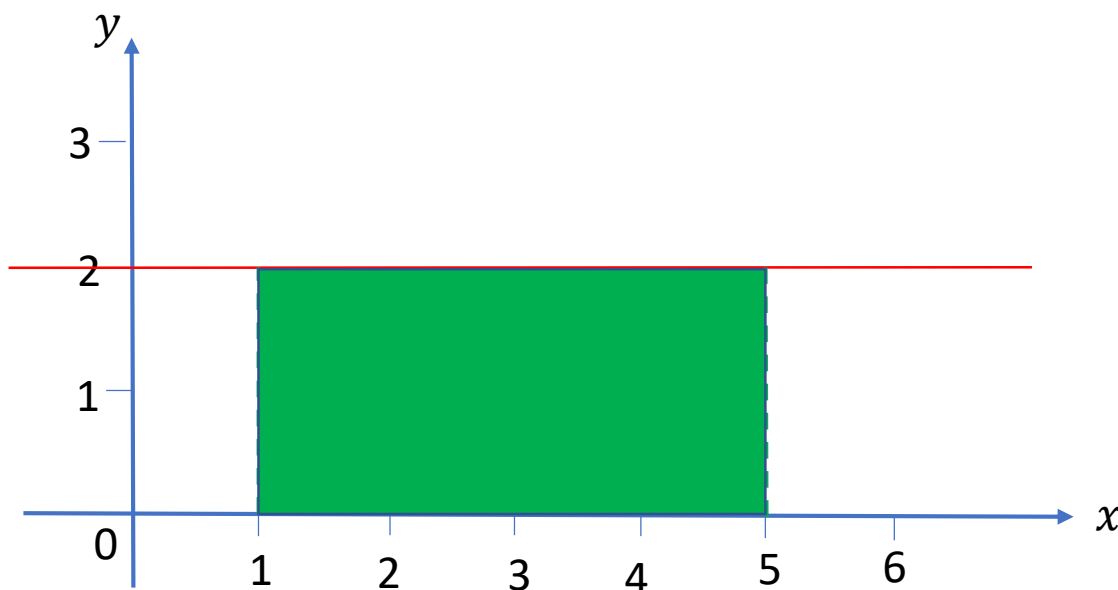
$$F(x) = \int_a^b f(x) dx$$

Example: $f(x) = 2$



Definite integral:

$$\int_1^5 2 dx = 2x \Big|_1^5 = 2(5 - 1) = 8$$



Area of the green shape:

$$\begin{aligned} A &= (5 - 1) \times (2 - 0) \\ &= 4 \times 2 \\ &= 8 \end{aligned}$$

Definite Integral (2)

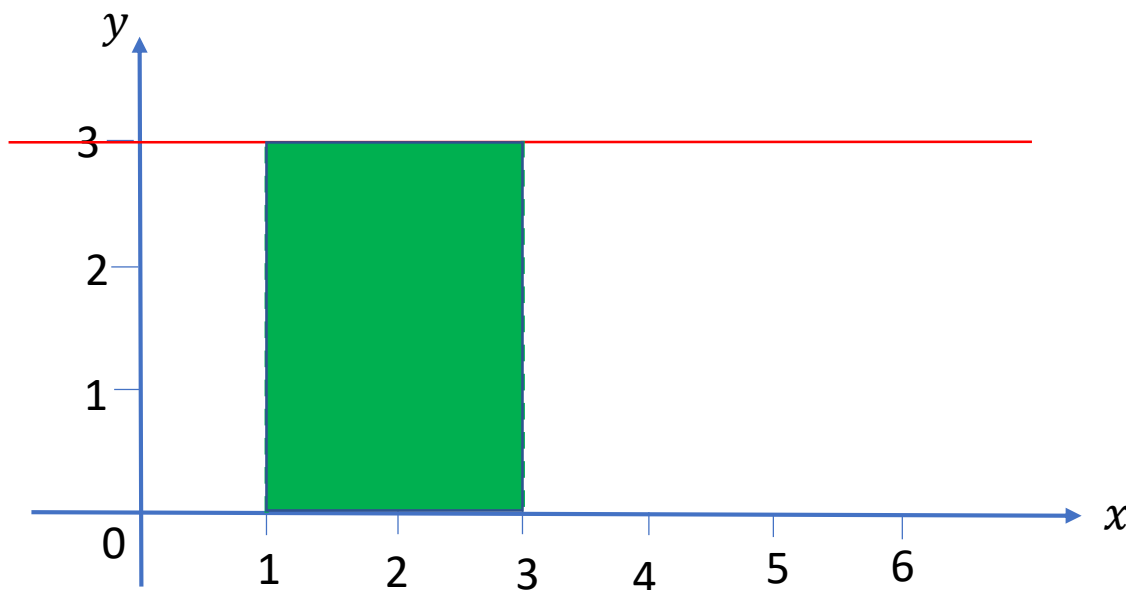
$$F(x) = \int_a^b f(x) dx$$

Example: $f(x) = 3$



Definite integral:

$$\int_1^3 3 dx = 3x \Big|_1^3 = 3(3 - 1) = 6$$



Area of the green shape:

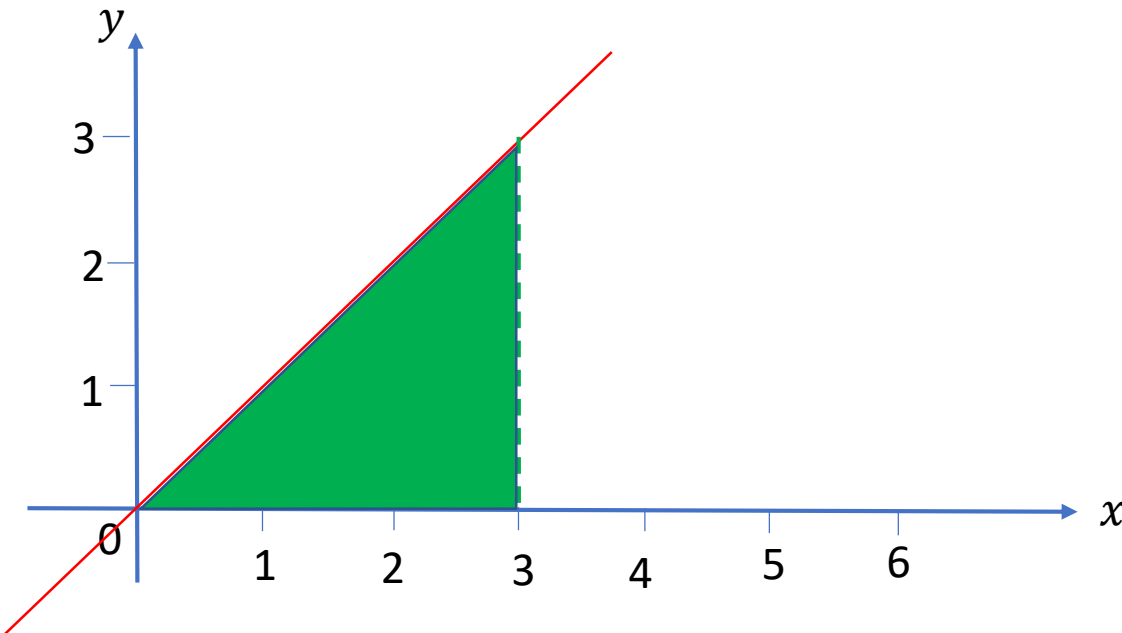
$$\begin{aligned} A &= (3 - 1) \times (3 - 0) \\ &= 2 \times 3 \\ &= 6 \end{aligned}$$

Definite Integral (3)

$$F(x) = \int_a^b f(x) dx$$

Example: $f(x) = x$  Definite integral:

$$\int_0^3 x dx = \frac{1}{2} x^2 \Big|_0^3 = \frac{1}{2} (3^2 - 0^2) = \frac{9}{2}$$



Area of the green shape:

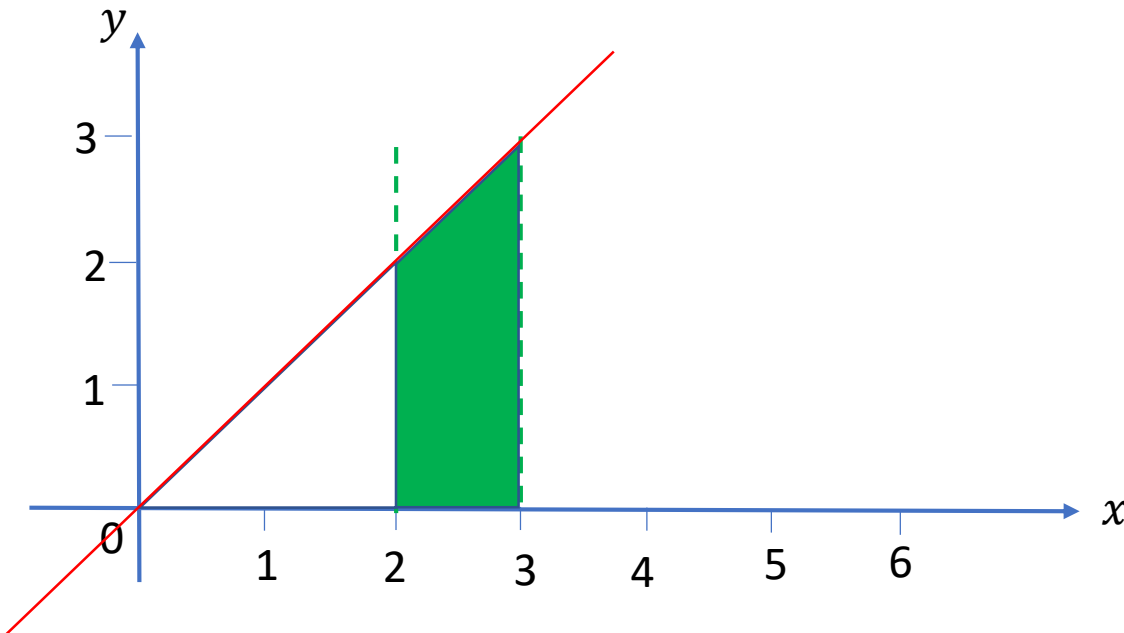
$$\begin{aligned} A &= \frac{1}{2} (3 - 0) \times (3) \\ &= \frac{1}{2} \times 3 \times 3 \\ &= \frac{9}{2} \end{aligned}$$

Definite Integral (4)

$$F(x) = \int_a^b f(x) dx$$

Example: $f(x) = x$  Definite integral:

$$\int_2^3 x dx = \frac{1}{2} x^2 \Big|_2^3 = \frac{1}{2} (3^2 - 2^2) = \frac{5}{2}$$



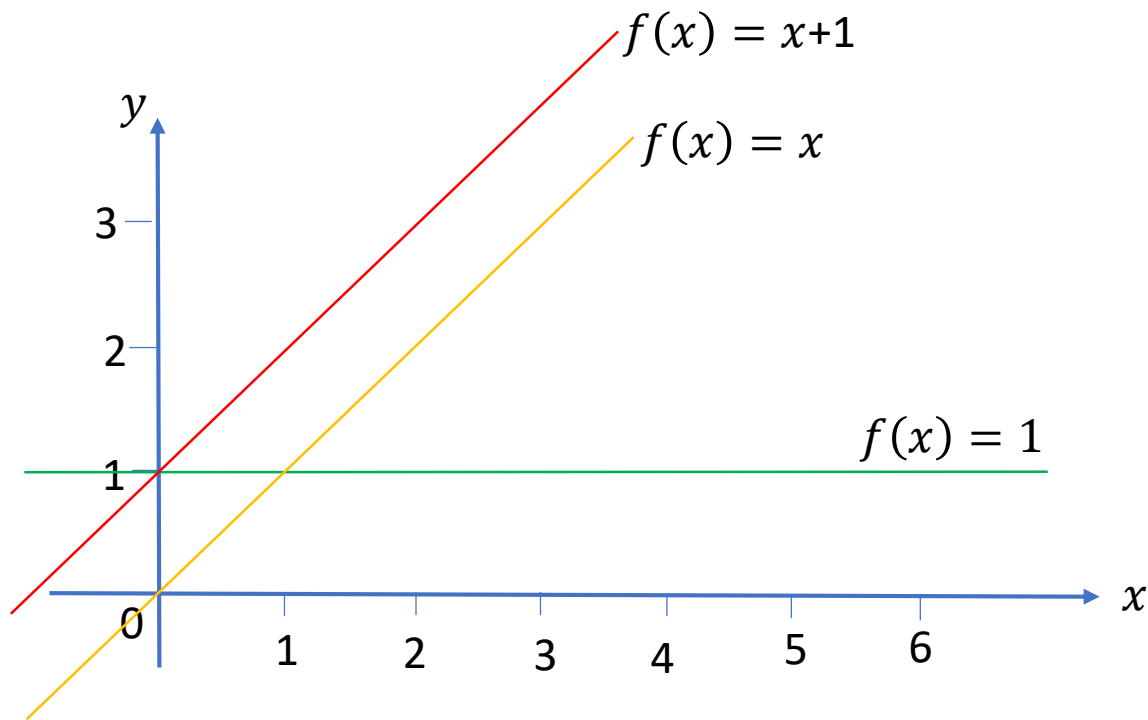
Area of the green shape:

$$\begin{aligned} A &= \frac{1}{2} (3 - 2) \times (2 + 3) \\ &= \frac{1}{2} \times 1 \times 5 \\ &= \frac{5}{2} \end{aligned}$$

Definite Integral (5)


$$F(x) = \int_a^b f(x) dx$$

Example: $f(x) = 1 + x$

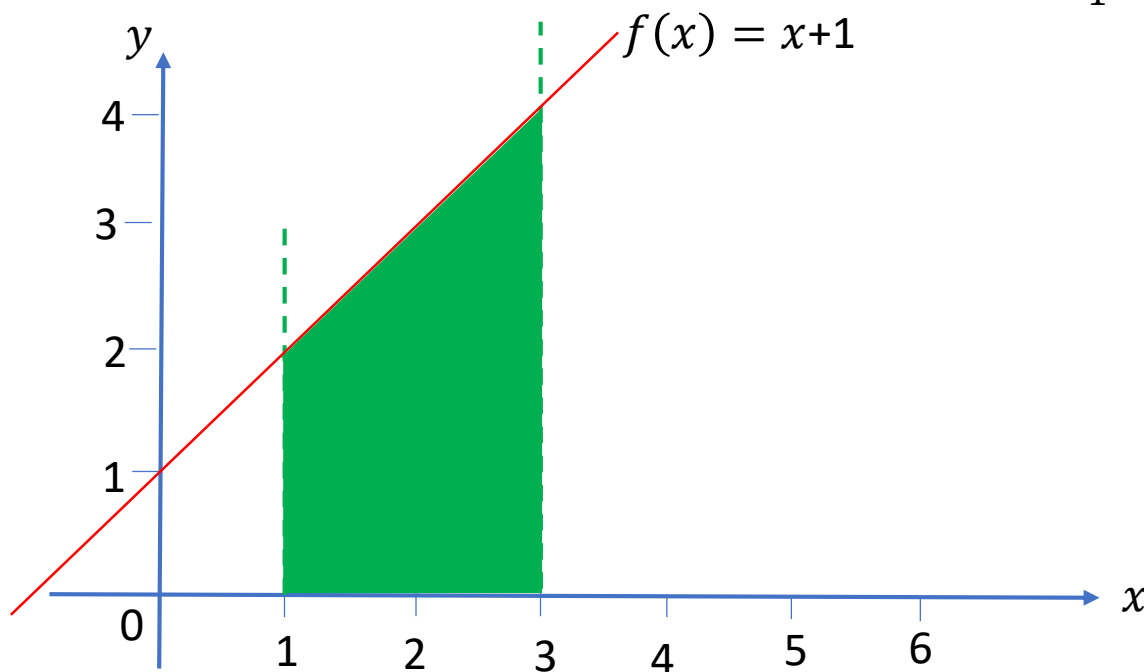


Definite Integral (5)

$$F(x) = \int_a^b f(x) dx$$

Example: $f(x) = 1 + x$  Definite integral:

$$\begin{aligned} \int_1^3 (1+x) dx &= \left(x + \frac{1}{2}x^2 \right) \Big|_1^3 \\ &= \left(3 + \frac{9}{2} \right) - \left(1 + \frac{1}{2} \right) = 6 \end{aligned}$$



Area of the green shape:

$$\begin{aligned} A &= \frac{1}{2} (3 - 1) \times (2 + 4) \\ &= \frac{1}{2} \times 2 \times 6 \\ &= 6 \end{aligned}$$

How to calculate area?

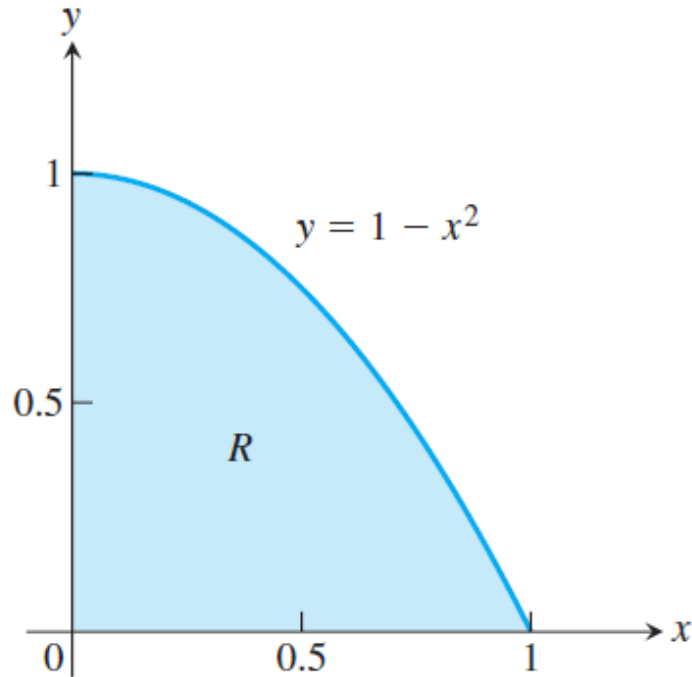


FIGURE 5.1 The area of the region R cannot be found by a simple formula.

How to calculate area?

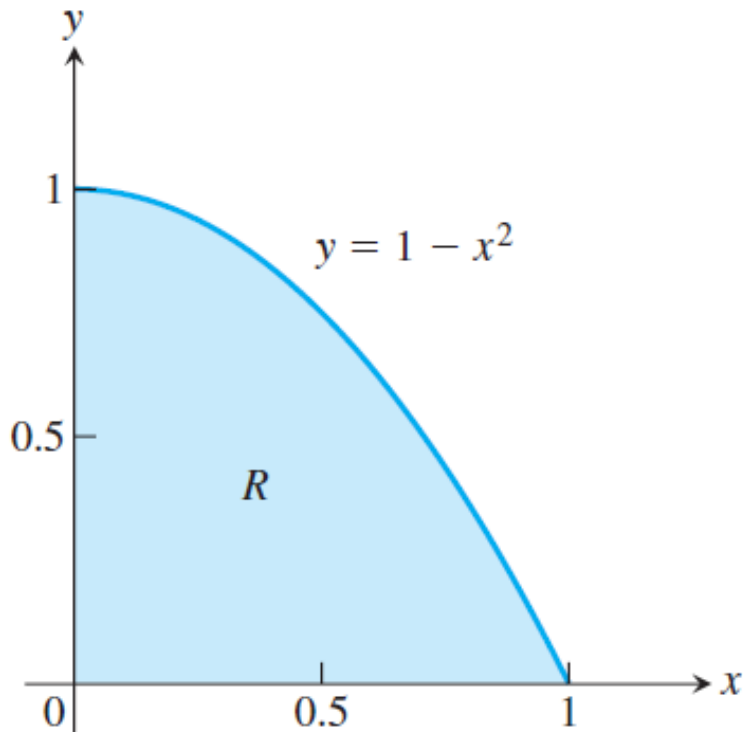
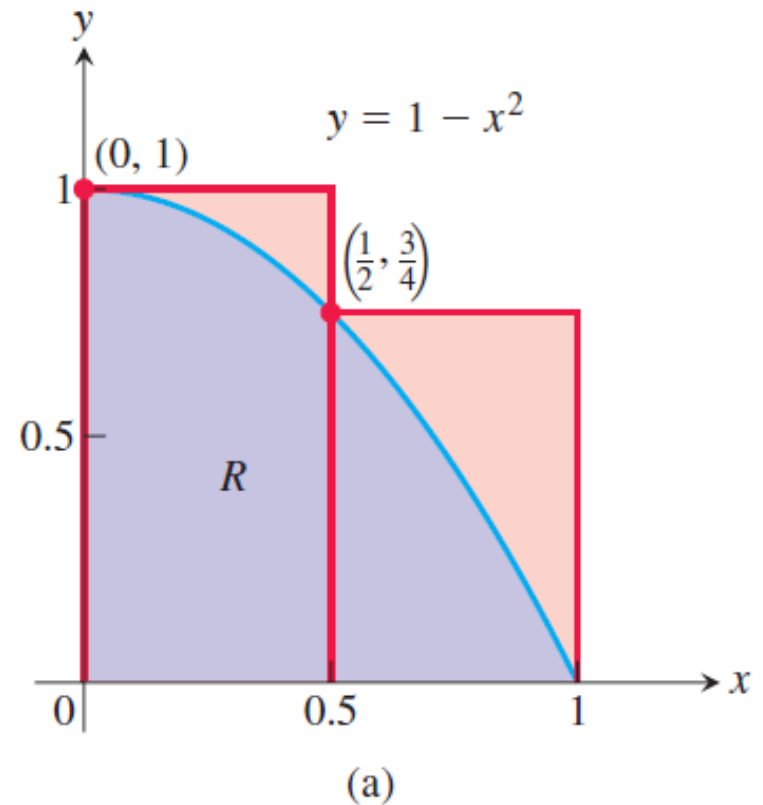


FIGURE 5.1 The area of the region R cannot be found by a simple formula.



$$A \approx 1 \cdot \frac{1}{2} + \frac{3}{4} \cdot \frac{1}{2} = \frac{7}{8} = 0.875.$$

How to calculate area?

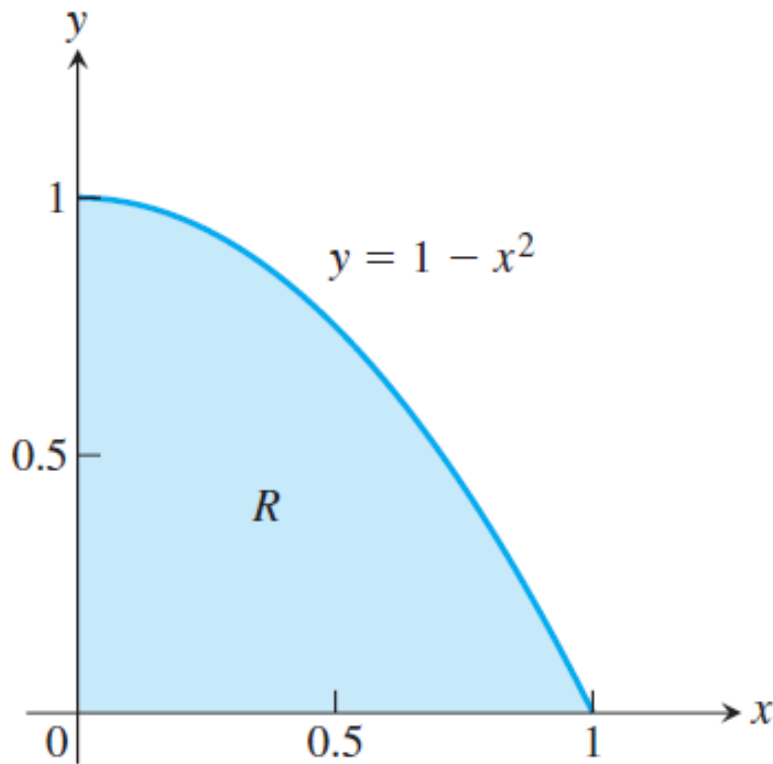
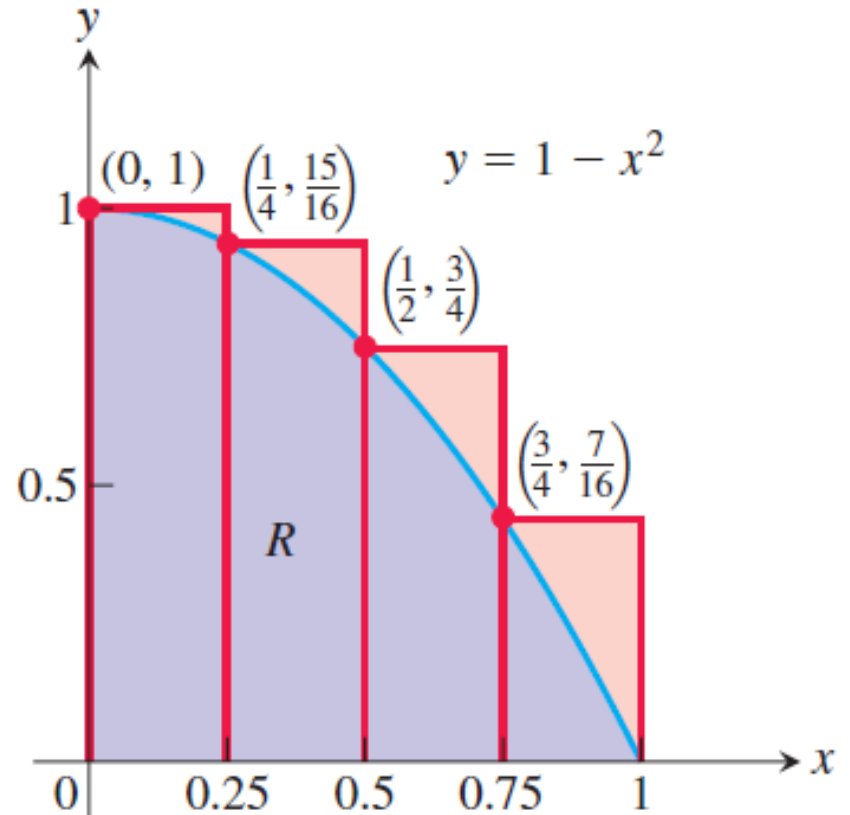


FIGURE 5.1 The area of the region R cannot be found by a simple formula.



$$A \approx 1 \cdot \frac{1}{4} + \frac{15}{16} \cdot \frac{1}{4} + \frac{3}{4} \cdot \frac{1}{4} + \frac{7}{16} \cdot \frac{1}{4} = \frac{25}{32} = 0.78125.$$

How to calculate area?

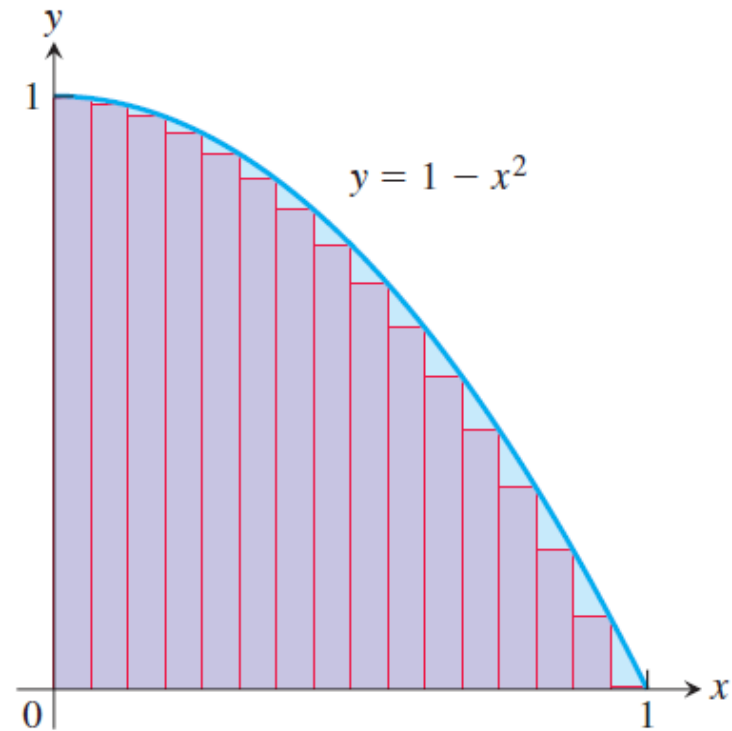
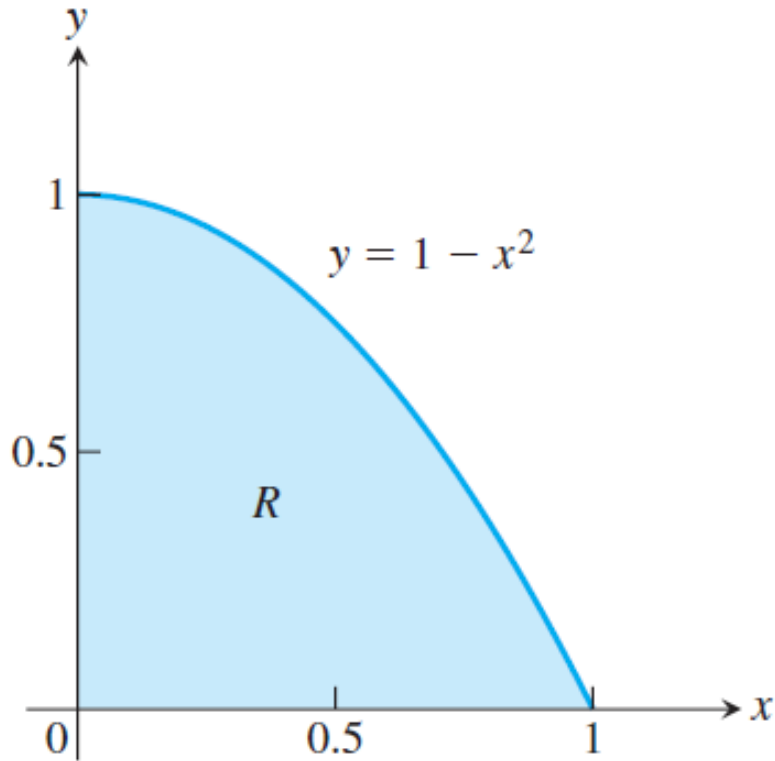


FIGURE 5.1 The area of the region R cannot be found by a simple formula.

0.634765625,

How to calculate area?

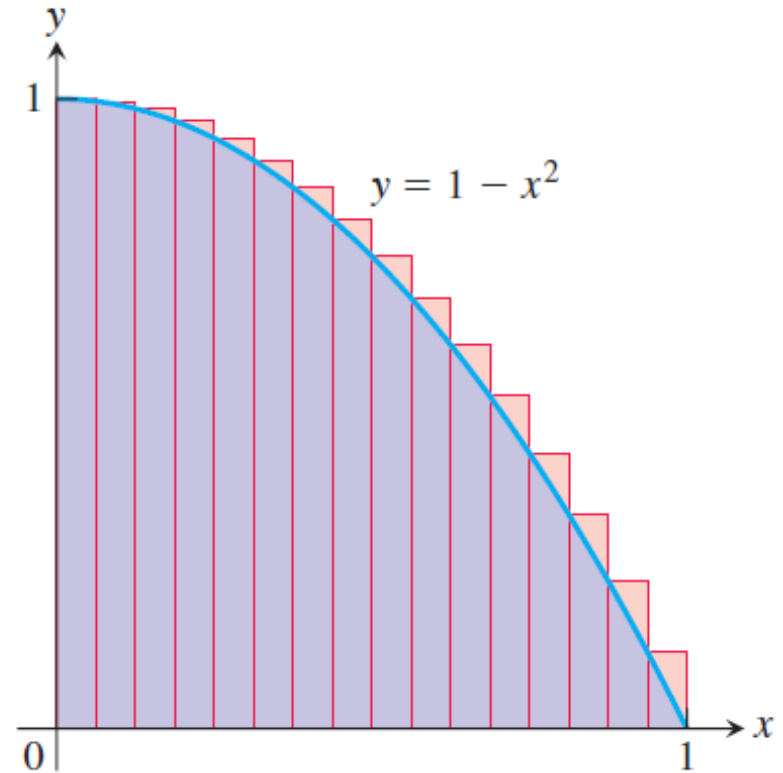
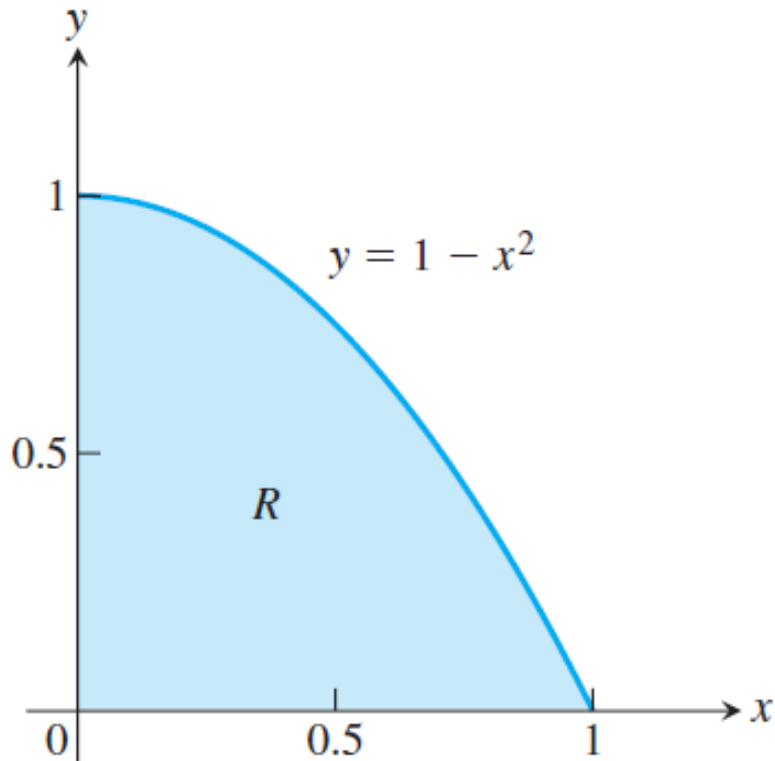


FIGURE 5.1 The area of the region R cannot be found by a simple formula.

0.697265625

Comparison in calculating area

