

Calculus 1

Lecture 2: Functions

By:

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Functions

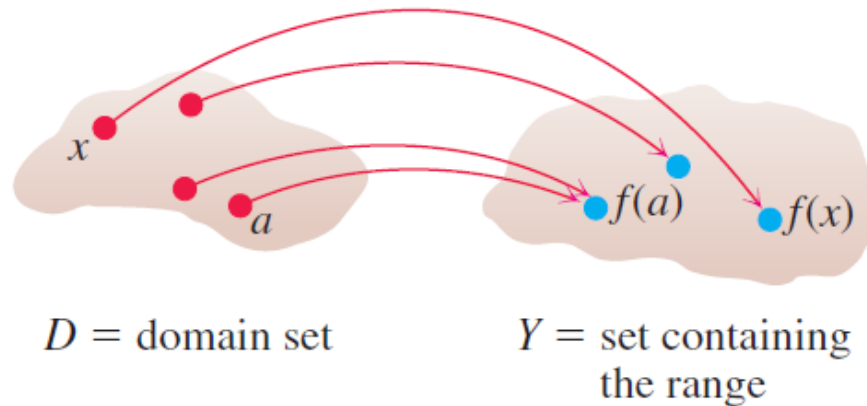
- Understanding function is a fundamental in study of calculus.
- A functions is a tool to describe the real world in mathematics.
- A function can be represented by an equation, a graph, a numerical table, or a verbal description.

Functions

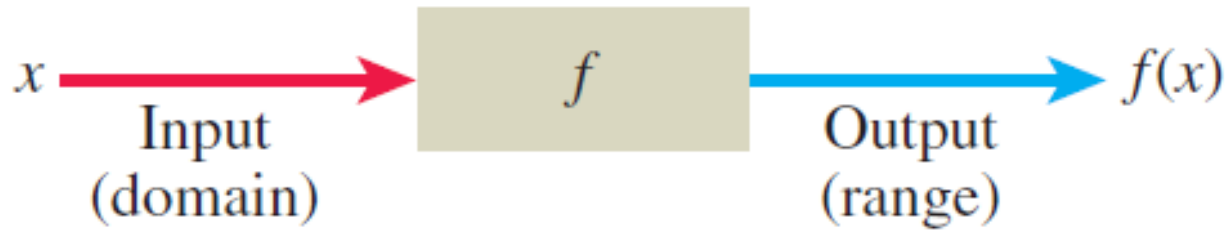
$$y = f(x) \quad (\text{"y equals } f \text{ of } x\text{").}$$

In this notation, the symbol f represents the function, the letter x is the **independent variable** representing the input value of f , and y is the **dependent variable** or output value of f at x .

DEFINITION A function f from a set D to a set Y is a rule that assigns a *unique* (single) element $f(x) \in Y$ to each element $x \in D$.



Function is a machine



Domain and Range



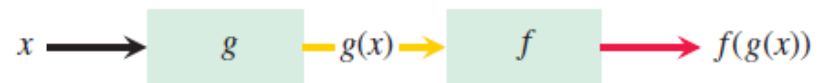
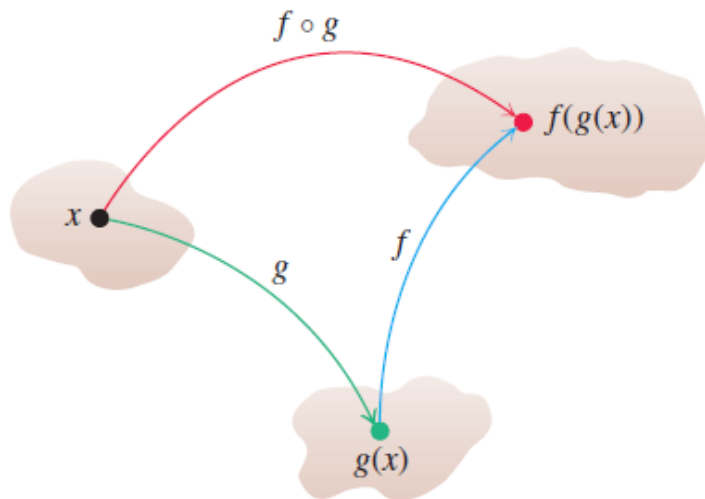
| Function | Domain (x) | Range (y) |
|----------------------|---------------------------------|---------------------------------|
| $y = x^2$ | $(-\infty, \infty)$ | $[0, \infty)$ |
| $y = 1/x$ | $(-\infty, 0) \cup (0, \infty)$ | $(-\infty, 0) \cup (0, \infty)$ |
| $y = \sqrt{x}$ | $[0, \infty)$ | $[0, \infty)$ |
| $y = \sqrt{4 - x}$ | $(-\infty, 4]$ | $[0, \infty)$ |
| $y = \sqrt{1 - x^2}$ | $[-1, 1]$ | $[0, 1]$ |

Composite Functions

DEFINITION If f and g are functions, the **composite** function $f \circ g$ (“ f composed with g ”) is defined by

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ consists of the numbers x in the domain of g for which $g(x)$ lies in the domain of f .



Graph of Function

If f is a function with domain D , its **graph** consists of the points in the Cartesian plane whose coordinates are the input-output pairs for f . In set notation, the graph is

$$\{(x, f(x)) \mid x \in D\}.$$

The graph of the function $f(x) = x + 2$ is the set of points with coordinates (x, y) for which $y = x + 2$. Its graph is the straight line sketched in Figure 1.3.

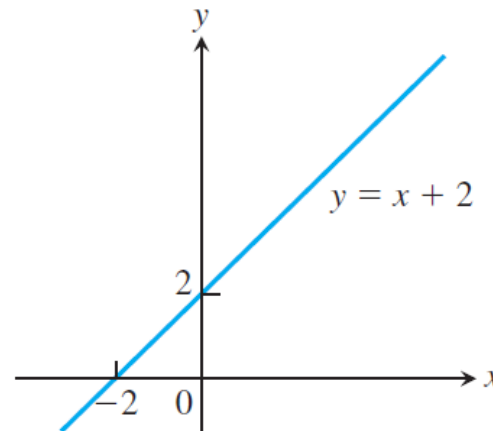


FIGURE 1.3 The graph of $f(x) = x + 2$ is the set of points (x, y) for which y has the value $x + 2$.

How to build a graph ?

EXAMPLE 2 Graph the function $y = x^2$ over the interval $[-2, 2]$.

Solution Make a table of xy -pairs that satisfy the equation $y = x^2$. Plot the points (x, y) whose coordinates appear in the table, and draw a *smooth* curve (labeled with its equation) through the plotted points (see Figure 1.5). ■

| x | $y = x^2$ |
|---------------|---------------|
| -2 | 4 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |
| $\frac{3}{2}$ | $\frac{9}{4}$ |
| 2 | 4 |

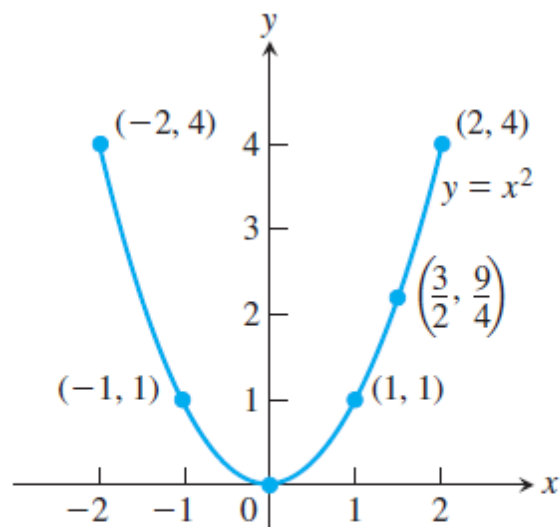


FIGURE 1.5 Graph of the function in Example 2.

Exercise

$$\begin{aligned}f(x) &= x + 2 \\g(x) &= x^2 - 4 \\y &= (f \circ g)(x)\end{aligned}$$

Exercise

Tentukan limit fungsi berikut secara analitik dan validasi hasil yang diperoleh dengan menggunakan grafik.

- $\lim_{x \rightarrow -1} \frac{x+1}{x^2-1}$

Exercise

- Tunjukkan secara analitik dan grafik bahwa fungsi $y = f(x) = x^2 + 2$ kontinu di $x = -1$.