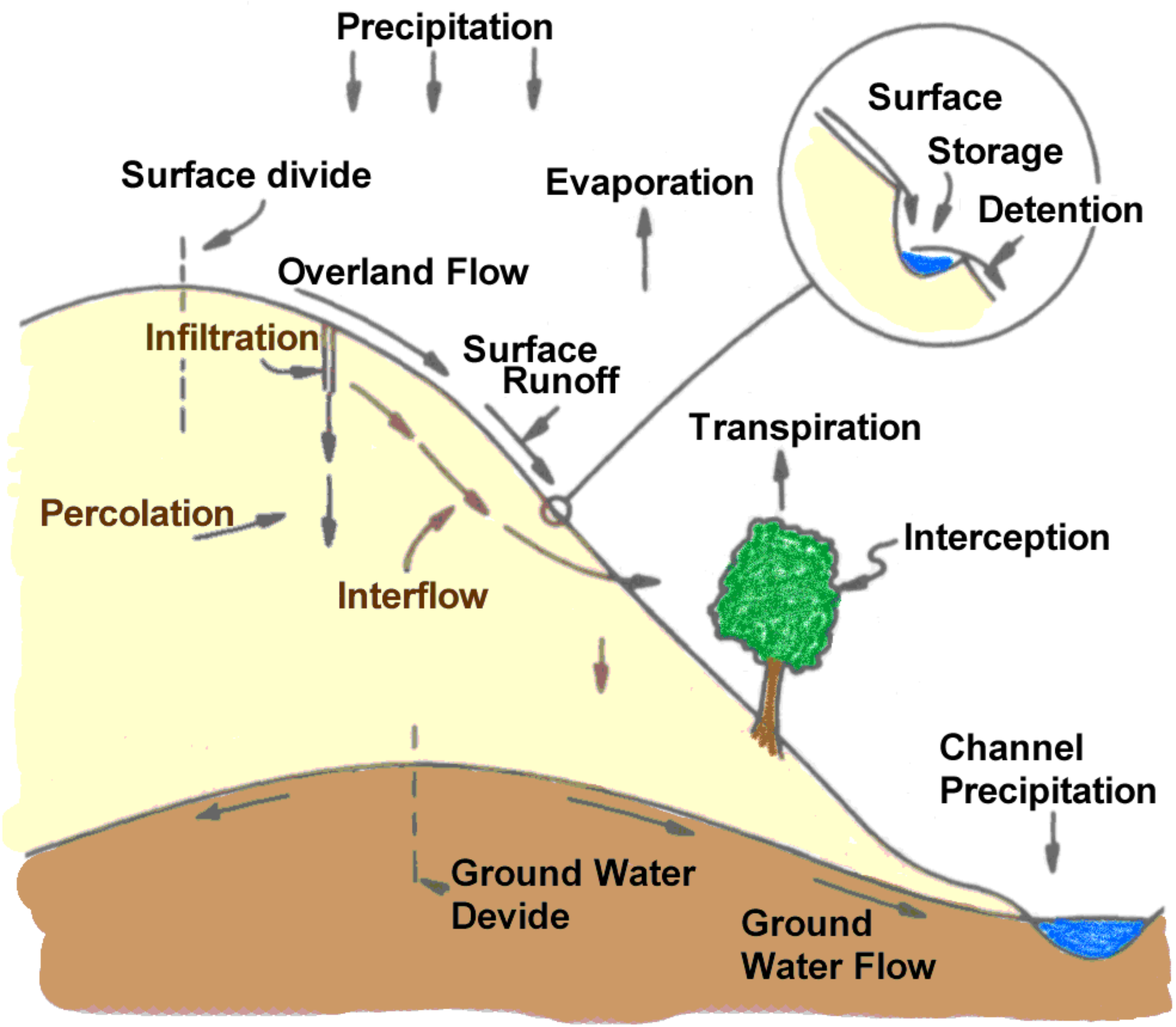
Unit Hydrograph

Unit Hydrograph Theory

- Sherman - 1932
- Horton - 1933
- Wisler & Brater - 1949 - “the hydrograph of surface runoff resulting from a relatively short, intense rain, called a unit storm.”
- The runoff hydrograph may be “made up” of runoff that is generated as flow through the soil (Black, 1990).

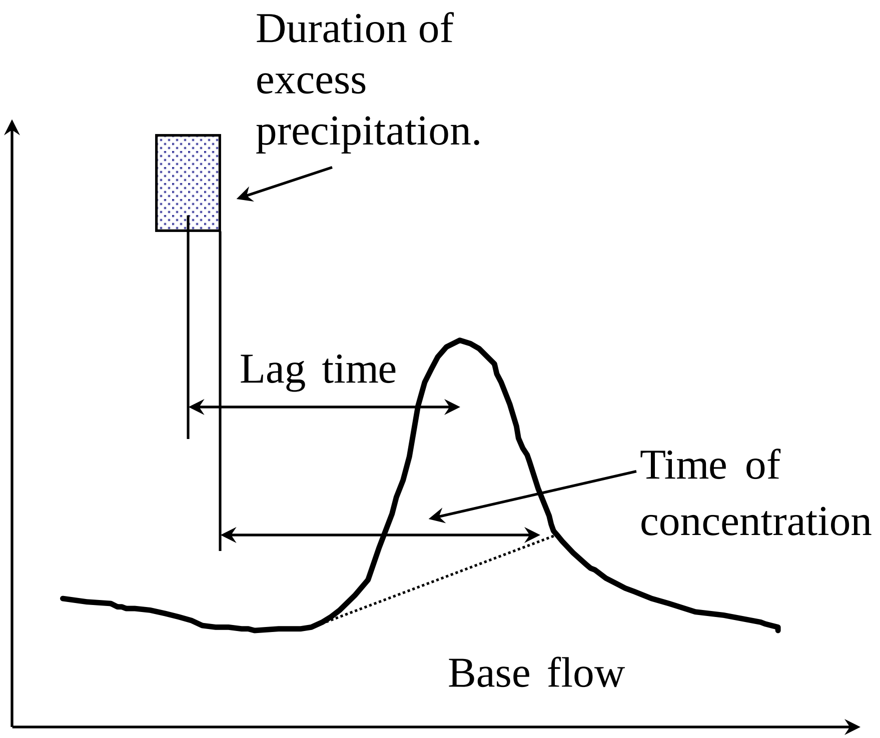
Unit Hydrograph Theory



Unit Hydrograph "Lingo"

- Duration
- Lag Time
- Time of Concentration
- Rising Limb
- Recession Limb (falling limb)
- Peak Flow
- Time to Peak (rise time)
- Recession Curve
- Separation
- Base flow

Graphical Representation



Methods of Developing UHG's

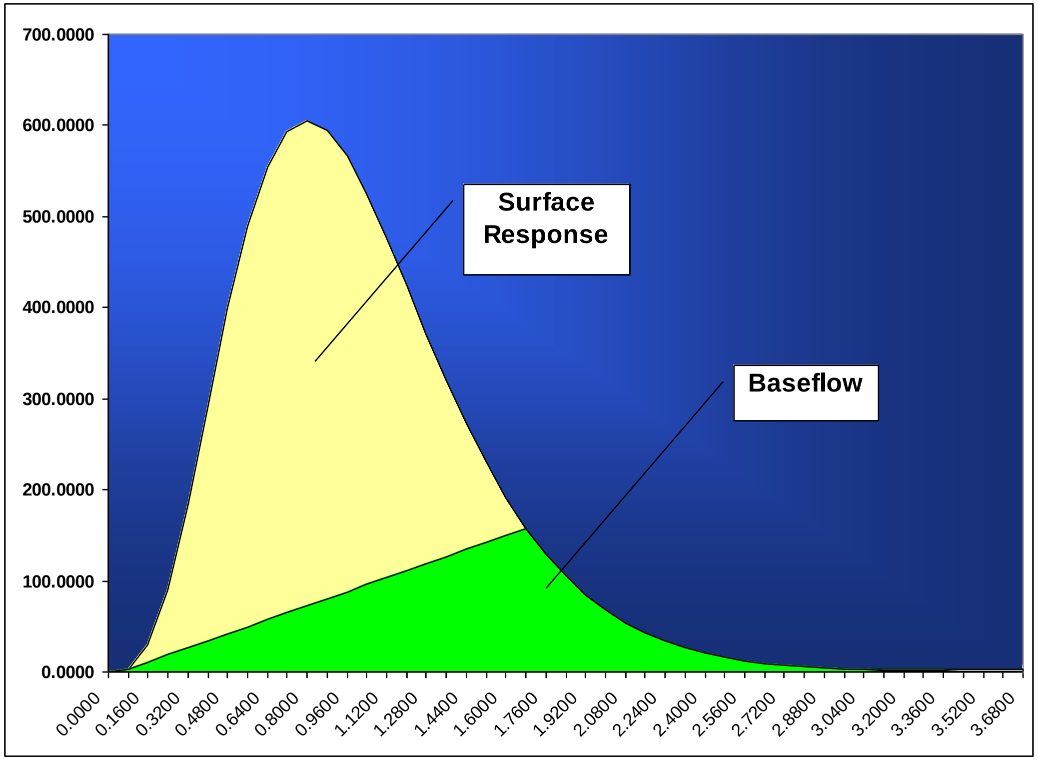
- From Streamflow Data
- Synthetically
  - Snyder
  - SCS
  - Time-Area (Clark, 1945)
- "Fitted" Distributions
- Geomorphologic

Unit Hydrograph

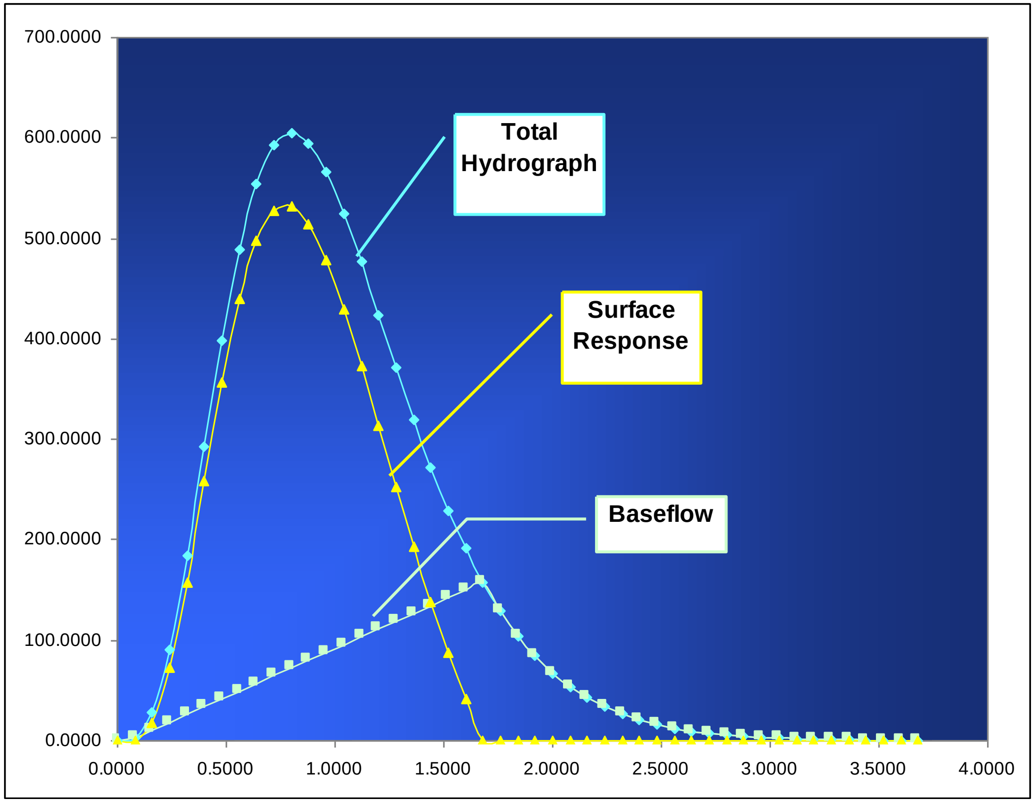
- ***The hydrograph that results from 1-inch of excess precipitation (or runoff) spread uniformly in space and time over a watershed for a given duration.***

- The key points :
  - ✓ 1-inch of EXCESS precipitation
  - ✓ Spread uniformly over space - evenly over the watershed
  - ✓ Uniformly in time - the excess rate is constant over the time interval
  - ✓ There is a given duration

Derived Unit Hydrograph



Derived Unit Hydrograph



Derived Unit Hydrograph

Rules of Thumb :

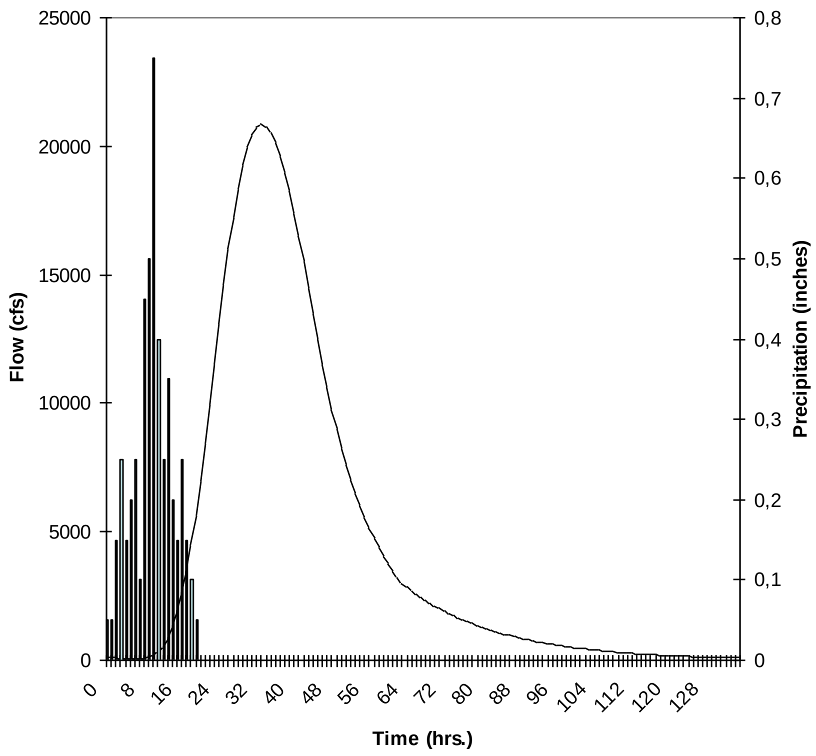
↘… the storm should be fairly uniform in nature and the excess precipitation should be equally as uniform throughout the basin. This may require the initial conditions throughout the basin to be spatially similar.

↘… Second, the storm should be relatively constant in time, meaning that there should be no breaks or periods of no precipitation.

↘… Finally, the storm should produce at least an inch of excess precipitation (the area under the hydrograph after correcting for baseflow).

Deriving a UHG from a Storm

sample watershed = 450 mi2



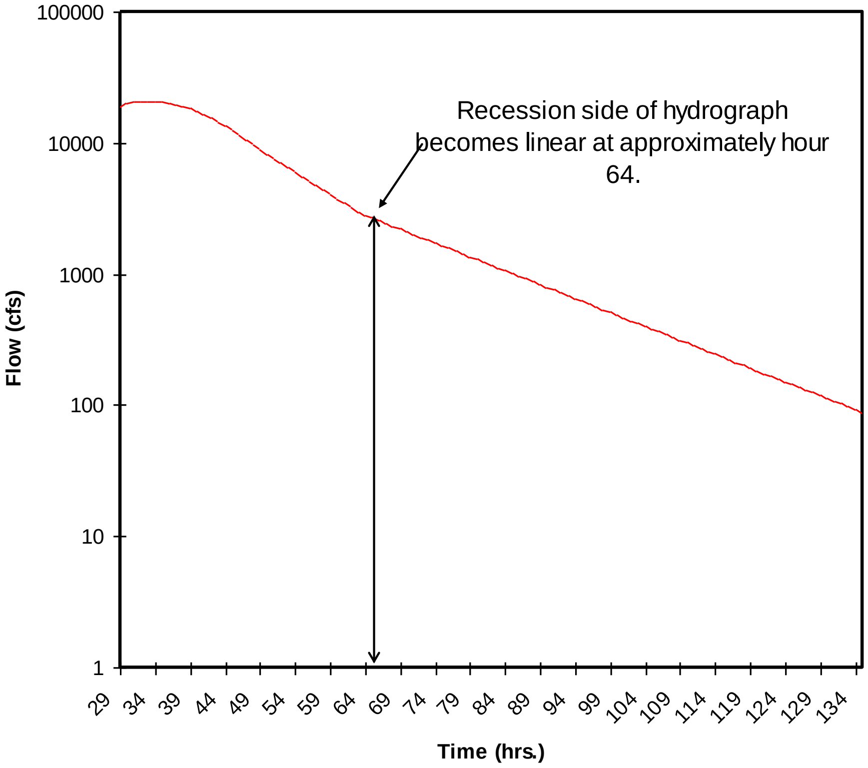
Separation of Baseflow

✓... generally accepted that the inflection point on the recession limb of a hydrograph is the result of a change in the controlling physical processes of the excess precipitation flowing to the basin outlet.

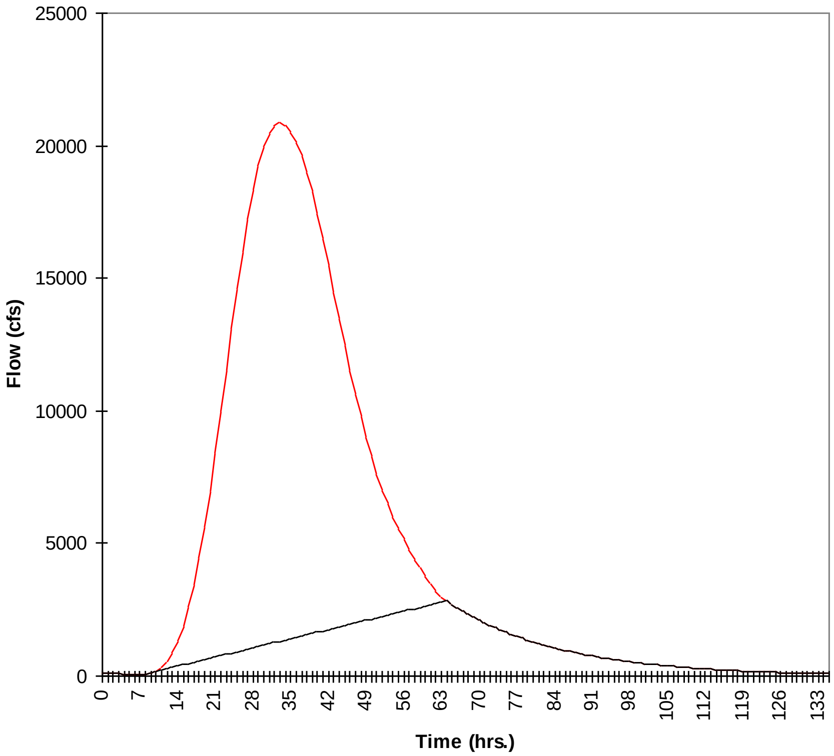
✓In this example, baseflow is considered to be a straight line connecting that point at which the hydrograph begins to rise rapidly and the inflection point on the recession side of the hydrograph.

✓the inflection point may be found by plotting the hydrograph in semi-log fashion with flow being plotted on the log scale and noting the time at which the recession side fits a straight line.

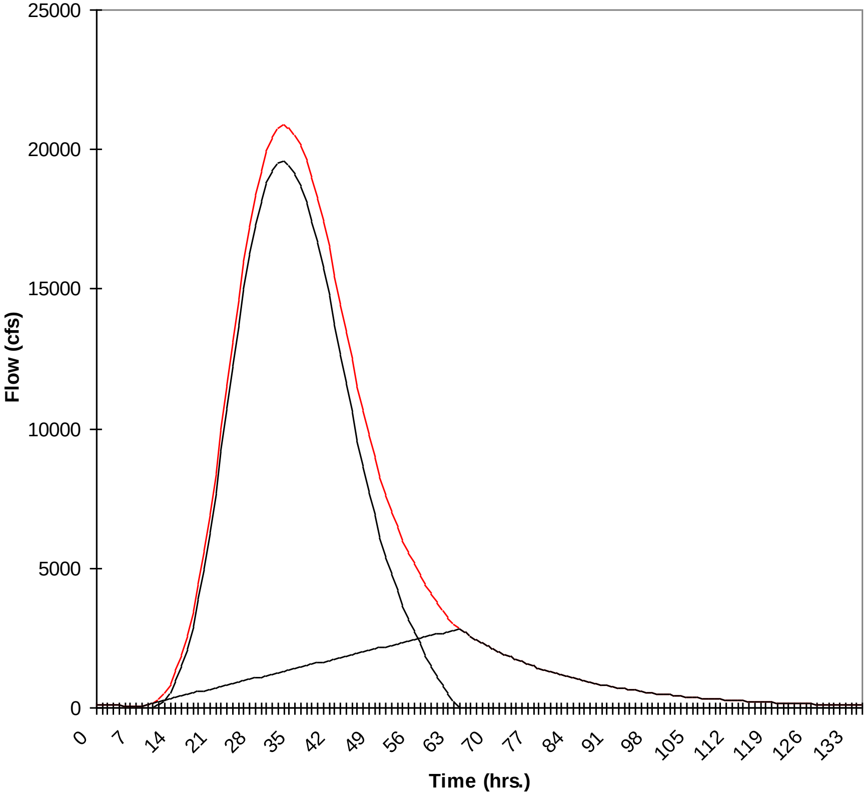
Semi-log Plot



Hydrograph & Baseflow



Separate Baseflow



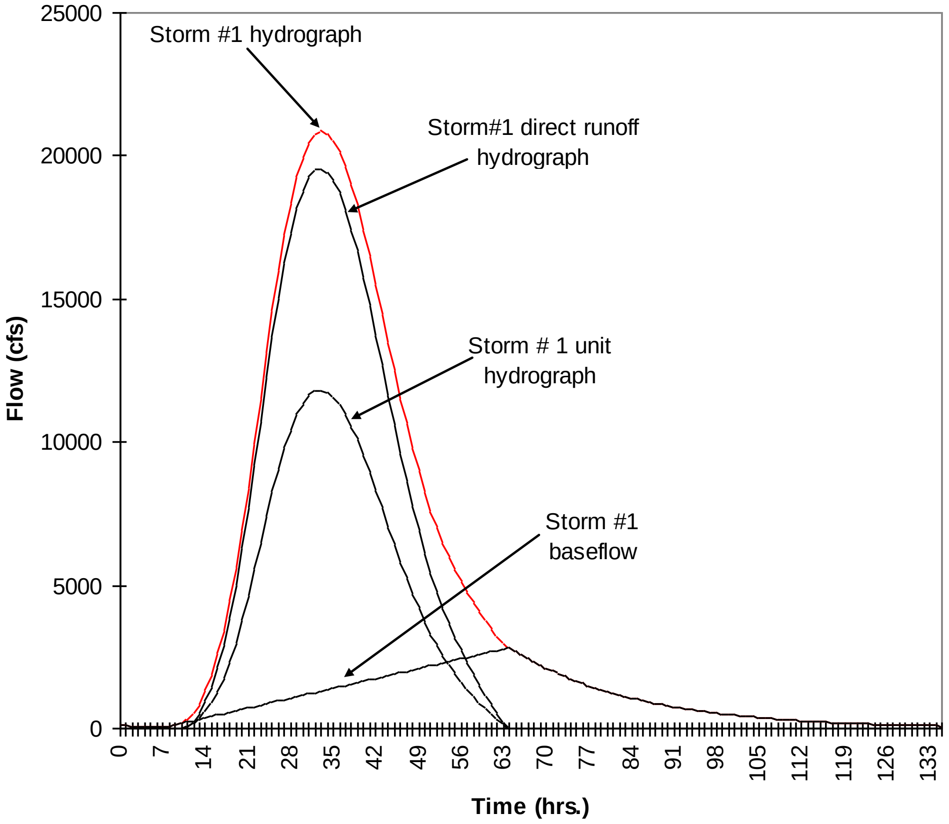
Sample Calculations

- In the present example (hourly time step), the flows are summed and then multiplied by 3600 seconds to determine the volume of runoff in cubic feet. If desired, this value may then be converted to acre-feet by dividing by 43,560 square feet per acre.
- The depth of direct runoff in feet is found by dividing the total volume of excess precipitation (now in acre-feet) by the watershed area (450 mi^2 converted to 288,000 acres).
- In this example, the volume of excess precipitation or direct runoff for storm #1 was determined to be 39,692 acre-feet.
- The depth of direct runoff is found to be 0.1378 feet after dividing by the watershed area of 288,000 acres.
- Finally, the depth of direct runoff in inches is 0.1378 x 12 = 1.65 inches.

Obtain UHG Ordinates

- The ordinates of the unit hydrograph are obtained by dividing each flow in the direct runoff hydrograph by the depth of excess precipitation.
- In this example, the units of the unit hydrograph would be cfs/inch (of excess precipitation).

Final UHG

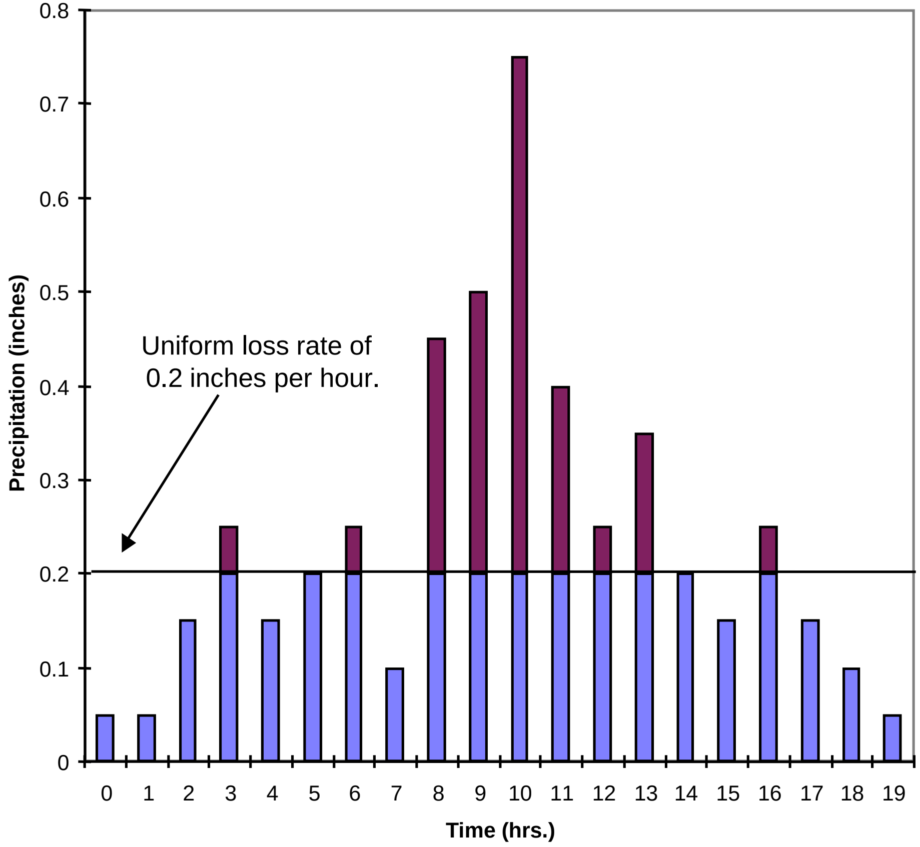


Storm #1 hydrograph
Storm#1 direct runoff hydrograph
Storm # 1 unit hydrograph
Storm #1 baseflow
Flow (cfs)
25000
20000
15000
10000
5000
0
0
7
14
21
28
35
42
49
56
63
70
77
84
91
98
105
112
119
126
133
Time (hrs.)

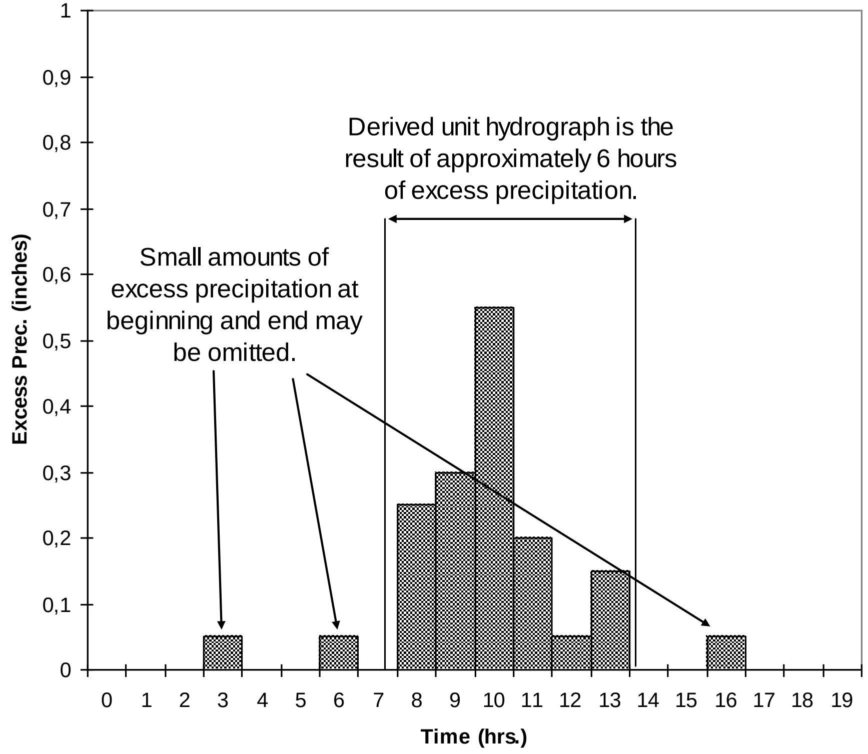
Determine Duration of UHG

- The duration of the derived unit hydrograph is found by examining the precipitation for the event and determining that precipitation which is in excess.
- This is generally accomplished by plotting the precipitation in hyetograph form and drawing a horizontal line such that the precipitation above this line is equal to the depth of excess precipitation as previously determined.
- This horizontal line is generally referred to as the Φ-index and is based on the assumption of a constant or uniform infiltration rate.
- The uniform infiltration necessary to cause 1.65 inches of excess precipitation was determined to be approximately 0.2 inches per hour.

Estimating Excess Precip.



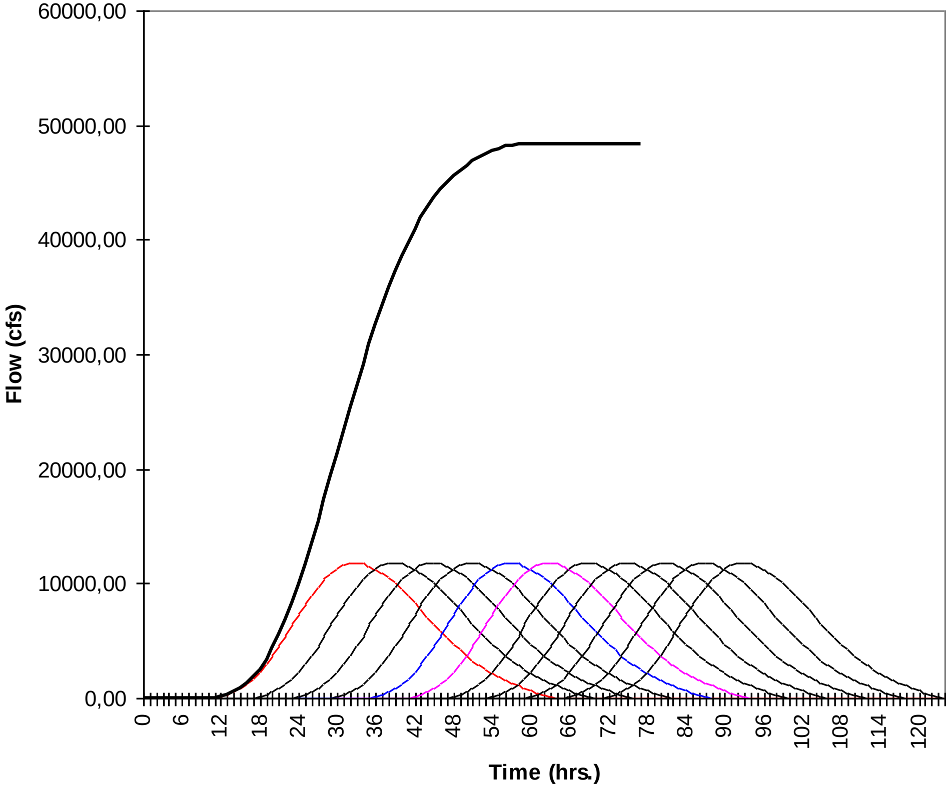
Excess Precipitation



Changing the Duration

- Very often, it will be necessary to change the duration of the unit hydrograph.
- If unit hydrographs are to be averaged, then they must be of the same duration.
- Also, convolution of the unit hydrograph with a precipitation event requires that the duration of the unit hydrograph be equal to the time step of the incremental precipitation.
- The most common method of altering the duration of a unit hydrograph is by the S-curve method.
- The S-curve method involves continually lagging a unit hydrograph by its duration and adding the ordinates.
- For the present example, the 6-hour unit hydrograph is continually lagged by 6 hours and the ordinates are added.

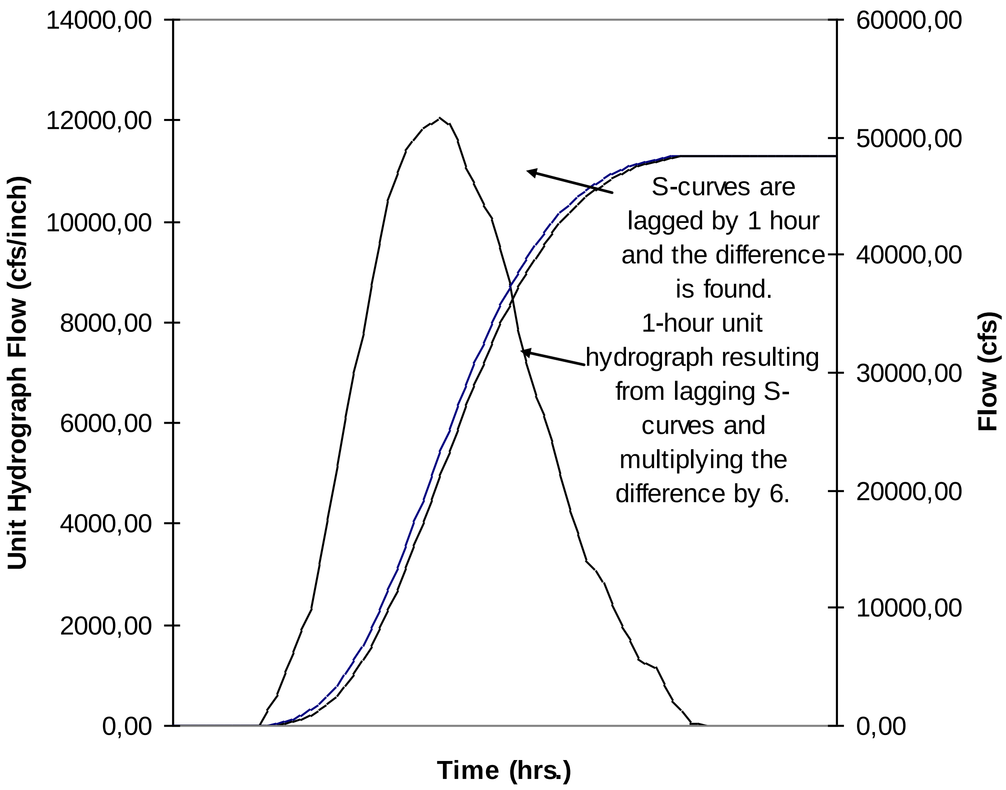
Develop S-Curve



Convert to 1-Hour Duration

- To arrive at a 1-hour unit hydrograph, the S-curve is lagged by 1 hour and the difference between the two lagged S-curves is found to be a 1 hour unit hydrograph.
- However, because the S-curve was formulated from unit hydrographs having a 6 hour duration of uniformly distributed precipitation, the hydrograph resulting from the subtracting the two S-curves will be the result of 1/6 of an inch of precipitation.
- Thus the ordinates of the newly created 1-hour unit hydrograph must be multiplied by 6 in order to be a true unit hydrograph.
- The 1-hour unit hydrograph should have a higher peak which occurs earlier than the 6-hour unit hydrograph.

Final 1-hour UHG



Shortcut Method

•There does exist a shortcut method for changing the duration of the unit hydrograph if the two durations are multiples of one another.

•This is done by displacing the the unit hydrograph.

•For example, if you had a two hour unit hydrograph and you wanted to change it to a four hour unit hydrograph.

Shortcut Method Example

•First, a two hour unit hydrograph is given and a four hour unit hydrograph is needed.

•There are two possiblities, develop the S - curve or since they are multiples use the shortcut method.

Time (hr)	Q
0	0
1	2
2	4
3	6
4	10
5	6
6	4
7	3
8	2
9	1
10	0

Shortcut Method Example

•The 2 hour UHG is then displaced by two hours. This is done because two 2 hour UHG will be used to represent a four hour UHG.

Time (hr)	Q	Displaced UHG
0	0	
1	2	
2	4	0
3	6	2
4	10	4
5	6	6
6	4	10
7	3	6
8	2	4
9	1	3
10	0	2
11		1
12		0

Shortcut Method Example

- These two hydrographs are then summed.

Time (hr)	Q	Displaced UHG	Sum
0	0		0
1	2		2
2	4	0	4
3	6	2	8
4	10	4	14
5	6	6	12
6	4	10	14
7	3	6	9
8	2	4	6
9	1	3	4
10	0	2	2
11		1	1
12		0	0

Shortcut Method Example

•Finally the summed hydrograph is divided by two.
•This is done because when two unit hydrographs are added, the area under the curve is two units. This has to be reduced back to one unit of runoff.

Time (hr)	Q	Displaced UHG	Sum	4 hour UHG
0	0		0	0
1	2		2	1
2	4	0	4	2
3	6	2	8	4
4	10	4	14	7
5	6	6	12	6
6	4	10	14	7
7	3	6	9	4.5
8	2	4	6	3
9	1	3	4	2
10	0	2	2	1
11		1	1	0.5
12		0	0	0

Average Several UHG's

- It is recommend that several unit hydrographs be derived and averaged.
- The unit hydrographs must be of the same duration in order to be properly averaged.
- It is often not sufficient to simply average the ordinates of the unit hydrographs in order to obtain the final unit hydrograph. A numerical average of several unit hydrographs which are different "shapes" may result in an "unrepresentative" unit hydrograph.
- It is often recommended to plot the unit hydrographs that are to be averaged. Then an average or representative unit hydrograph should be sketched or fitted to the plotted unit hydrographs.
- Finally, the average unit hydrograph must have a volume of 1 inch of runoff for the basin.

Synthetic UHG's

- Snyder
- SCS
- Time-area
- IHABBS Implementation Plan :

NOHRSC Homepage

http://www.nohrsc.nws.gov/

http://www.nohrsc.nws.gov/98/html/uhg/index.html

Snyder

- Since peak flow and time of peak flow are two of the most important parameters characterizing a unit hydrograph, the Snyder method employs factors defining these parameters, which are then used in the synthesis of the unit graph (Snyder, 1938).
- The parameters are C_p, the peak flow factor, and C_t, the lag factor.
- The basic assumption in this method is that basins which have similar physiographic characteristics are located in the same area will have similar values of C_t and C_p.
- Therefore, for ungaged basins, it is preferred that the basin be near or similar to gaged basins for which these coefficients can be determined.

Basic Relationships

$$t_{LAG} = C_t (L \bullet L_{ca})^{0.3}$$

$$t_{duration} = {t_{LAG}}\big/{5.5}$$

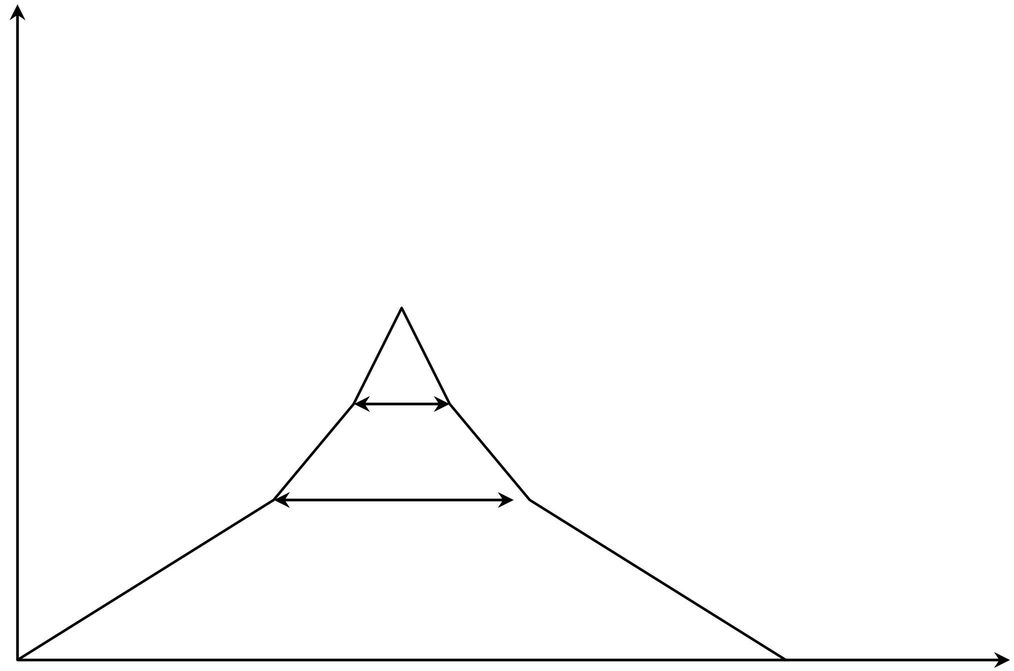
$$t_{alt.lag} = t_{LAG} + 0.25(t_{alt.duration} - t_{duration})$$

$$t_{base} = 3 + \frac{t_{LAG}}{8}$$

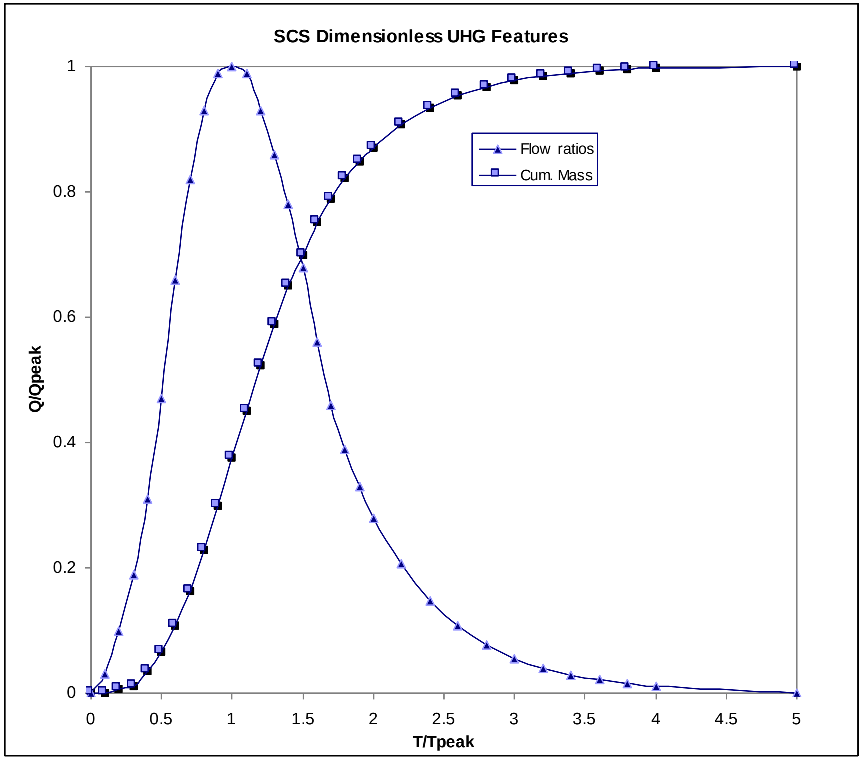
$$q_{peak} = \frac{640 A C_p}{t_{LAG}}$$

Final Shape

The final shape of the Snyder unit hydrograph is controlled by the equations for width at 50% and 75% of the peak of the UHG:



SCS

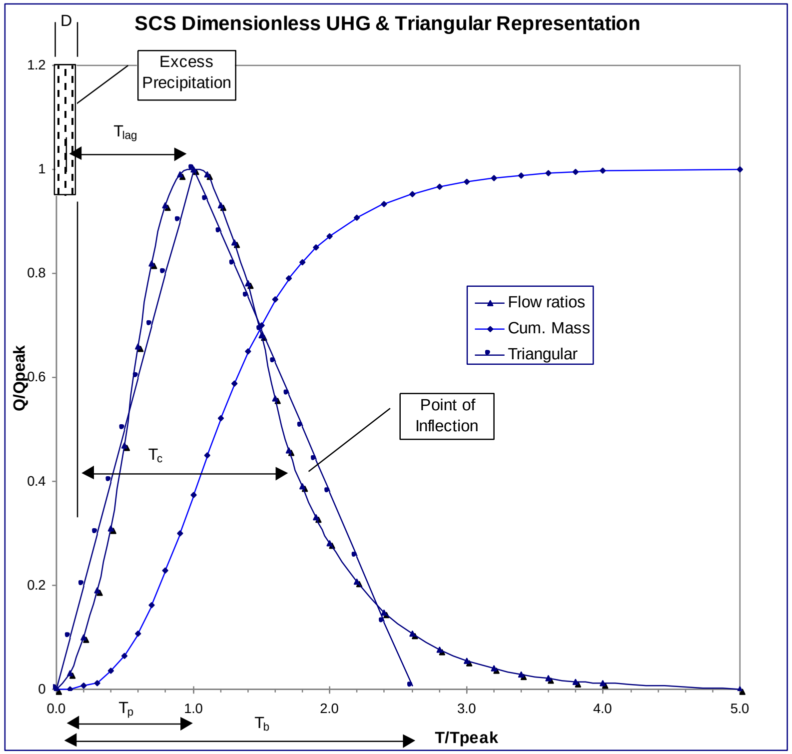


SCS Dimensionless UHG Features
Q/Qpeak
T/Tpeak
Flow ratios
Cum. Mass
0
0.2
0.4
0.6
0.8
1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Dimensionless Ratios

Time Ratios (t/t_p)	Discharge Ratios (q/q_p)	Mass Curve Ratios (Q_a/Q)
0	.000	.000
.1	.030	.001
.2	.100	.006
.3	.190	.012
.4	.310	.035
.5	.470	.065
.6	.660	.107
.7	.820	.163
.8	.930	.228
.9	.990	.300
1.0	1.000	.375
1.1	.990	.450
1.2	.930	.522
1.3	.860	.589
1.4	.780	.650
1.5	.680	.700
1.6	.560	.751
1.7	.460	.790
1.8	.390	.822
1.9	.330	.849
2.0	.280	.871
2.2	.207	.908
2.4	.147	.934
2.6	.107	.953
2.8	.077	.967
3.0	.055	.977
3.2	.040	.984
3.4	.029	.989
3.6	.021	.993
3.8	.015	.995
4.0	.011	.997
4.5	.005	.999
5.0	.000	1.000

Triangular Representation



Triangular Representation

$$T_b = 2.67 \text{ x } T_p$$

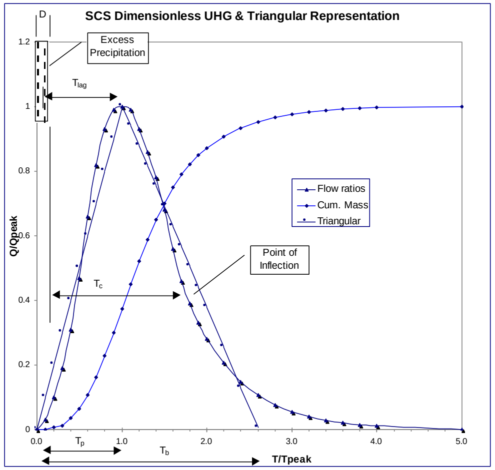
$$T_r = T_b - T_p = 1.67 \text{ x } T_p$$

$$Q = \frac{q_p T_p}{2} + \frac{q_p T_r}{2} = \frac{q_p}{2}(T_p + T_r)$$

$$q_p = \frac{2Q}{T_p + T_r}$$

$$q_p = \frac{654.33 x\, 2\, x\, A\, x\, Q}{T_p + T_r}$$

$$q_p = \frac{484\, A\, Q}{T_p}$$



The 645.33 is the conversion used for delivering 1-inch of runoff (the area under the unit hydrograph) from 1-square mile in 1-hour (3600 seconds).

484 ?

$$q_p = \frac{484\,A\,Q}{T_p}$$

Comes from the initial assumption that 3/8 of the volume under the UHG is under the rising limb and the remaining 5/8 is under the recession limb.

General Description	Peaking Factor	Limb Ratio (Recession to Rising)
Urban areas; steep slopes	575	1.25
Typical SCS	484	1.67
Mixed urban/rural	400	2.25
Rural, rolling hills	300	3.33
Rural, slight slopes	200	5.5
Rural, very flat	100	12.0

Duration & Timing?

Again from the triangle

$$T_p = \frac{D}{2} + L$$

L = Lag time

$$L = 0.6 * T_c$$

$$T_c + D = 1.7\,T_p$$

$$\frac{D}{2} + 0.6\,T_c = T_p$$

For estimation purposes : $D = 0.133\,T_c$

Time of Concentration

- Regression Eqs.
- Segmental Approach

A Regression Equation

$$T_{lag} = \frac{L^{0.8}(S+1)^{0.7}}{1900(\%Slope)^{0.5}}$$

where : T_{lag} = lag time in hours
L = Length of the longest drainage path in feet
S = (1000/CN) - 10 (CN=curve number)
%Slope = The average watershed slope in %

Segmental Approach

- More “hydraulic” in nature
- The parameter being estimated is essentially the time of concentration or longest travel time within the basin.
- In general, the longest travel time corresponds to the longest drainage path
- The flow path is broken into segments with the flow in each segment being represented by some type of flow regime.
- The most common flow representations are overland, sheet, rill and gully, and channel flow.

A Basic Approach

$$V = kS^{\frac{1}{2}}$$

K	**Land Use / Flow Regime**
0.25	Forest with heavy ground litter, hay meadow (overland flow)
0.5	Trash fallow or minimum tillage cultivation; contour or strip cropped; woodland (overland flow)
0.7	Short grass pasture (overland flow)
0.9	Cultivated straight row (overland flow)
1.0	Nearly bare and untilled (overland flow); alluvial fans in western mountain regions
1.5	Grassed waterway
2.0	Paved area (sheet flow); small upland gullies

McCuen (1989) and SCS (1972) provide values of k for several flow situations (slope in %)

Flow Type	**K**
Small Tributary - Permanent or intermittent streams which appear as solid or dashed blue lines on USGS topographic maps.	2.1
Waterway - Any overland flow route which is a well defined swale by elevation contours, but is not a stream section as defined above.	1.2
Sheet Flow - Any other overland flow path which does not conform to the definition of a waterway.	0.48

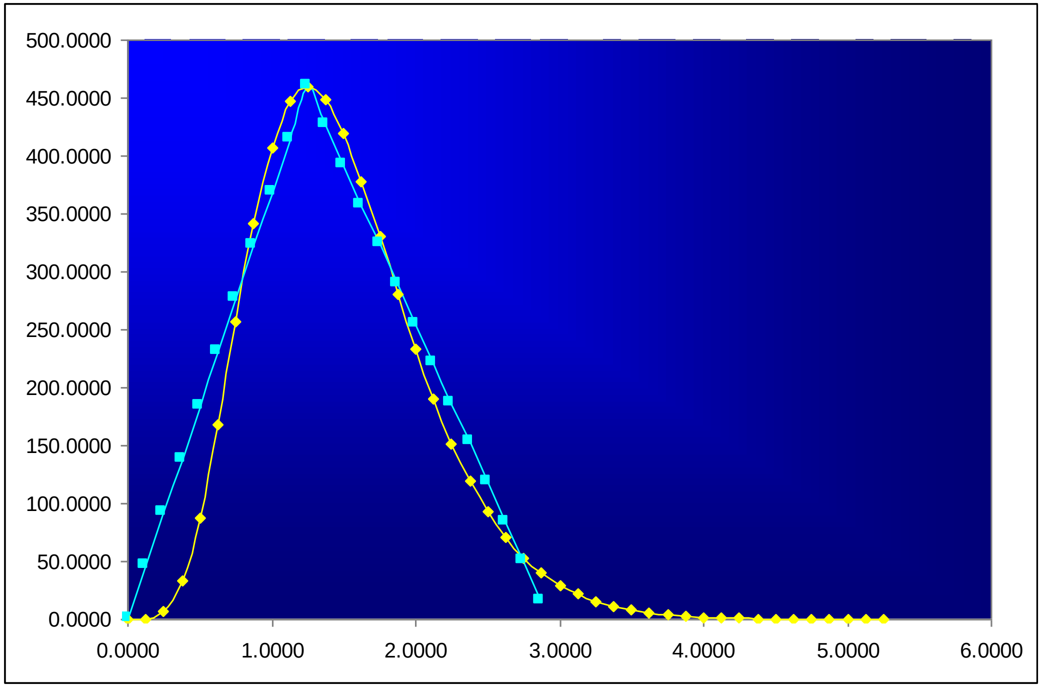
Sorell & Hamilton, 1991

Triangular Shape

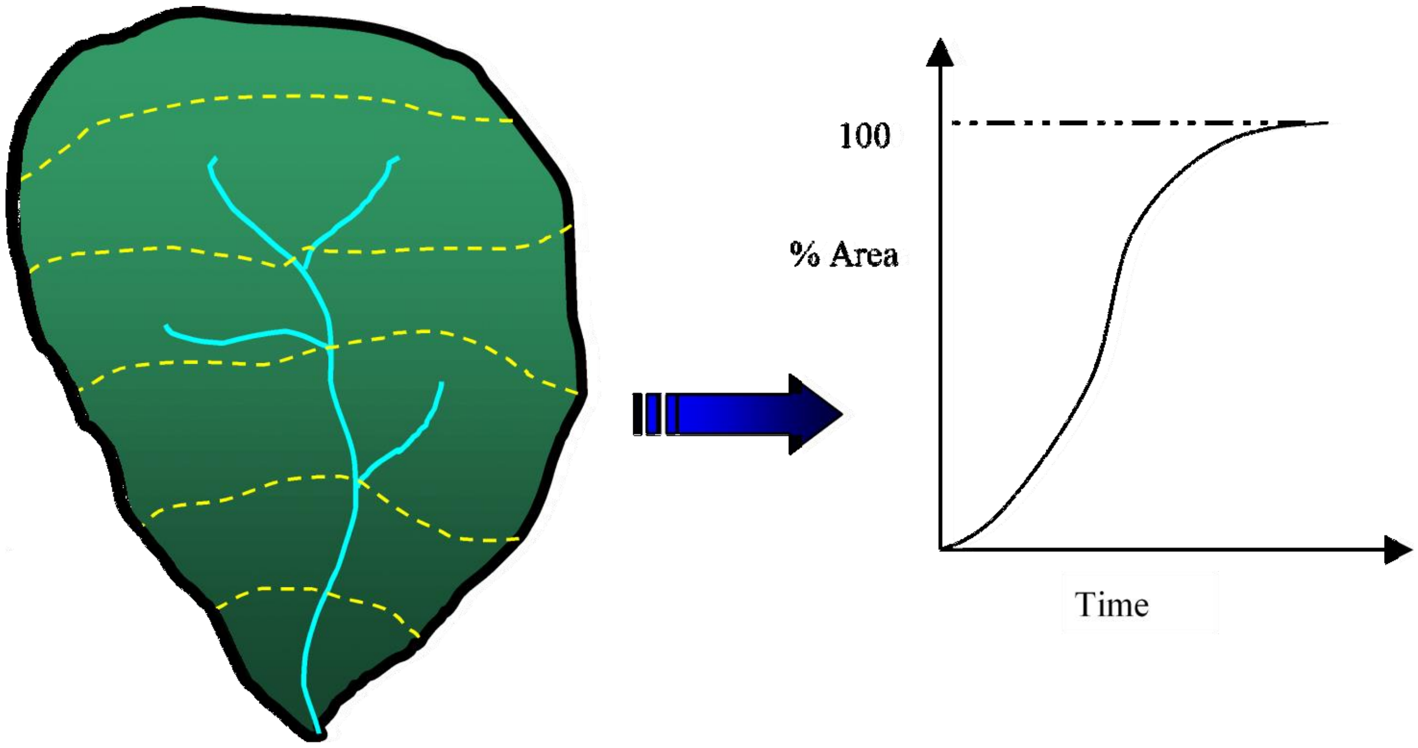
- In general, it can be said that the triangular version will not cause or introduce noticeable differences in the simulation of a storm event, particularly when one is concerned with the peak flow.
- For long term simulations, the triangular unit hydrograph does have a potential impact, due to the shape of the recession limb.
- The U.S. Army Corps of Engineers (HEC 1990) fits a Clark unit hydrograph to match the peak flows estimated by the Snyder unit hydrograph procedure.
- It is also possible to fit a synthetic or mathematical function to the peak flow and timing parameters of the desired unit hydrograph.
- Aron and White (1982) fitted a gamma probability distribution using peak flow and time to peak data.

Fitting a Gamma Distribution

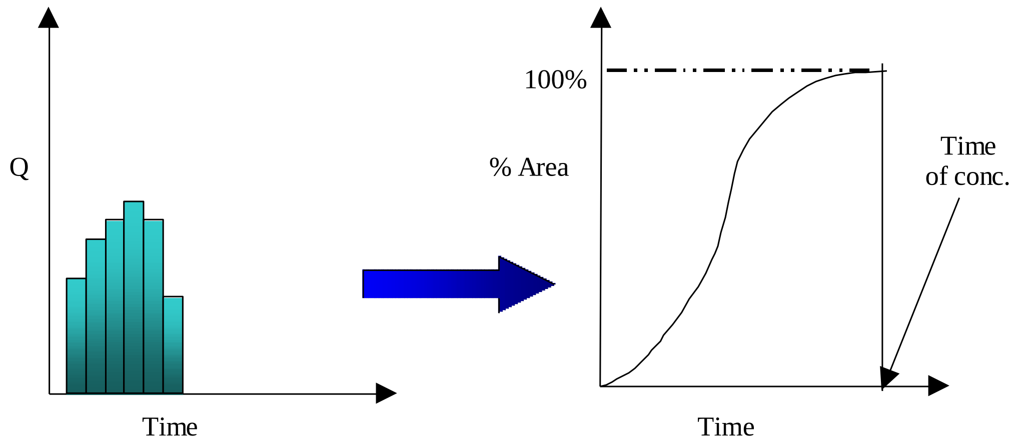
$$f(t;a,b)=\frac{t^{a}e^{-t/b}}{b^{a+1}\Gamma(a+1)}$$



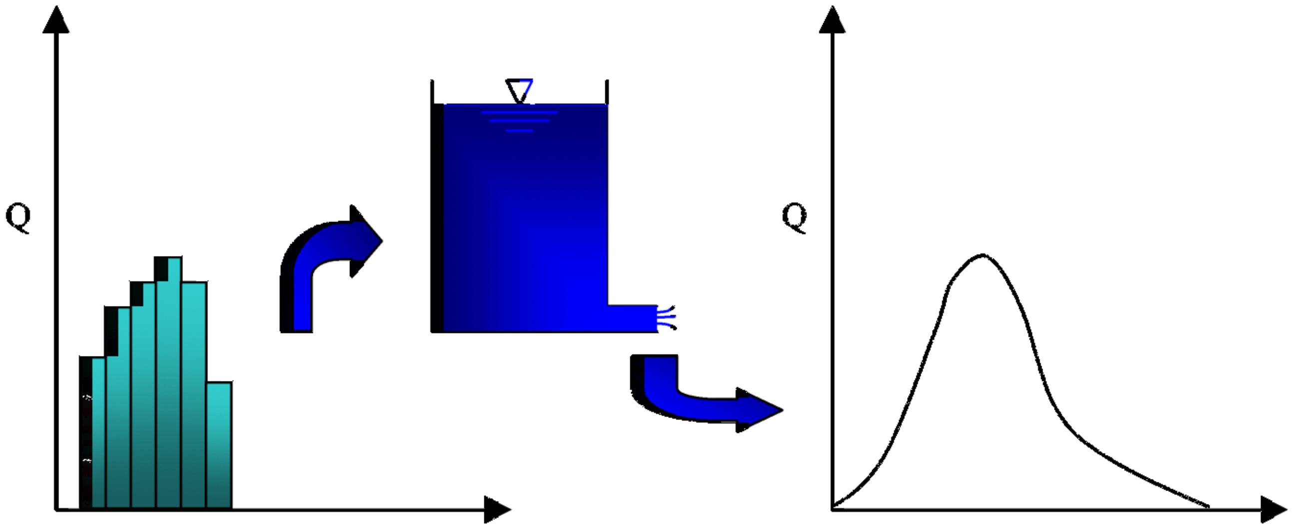
Time-Area



Time-Area

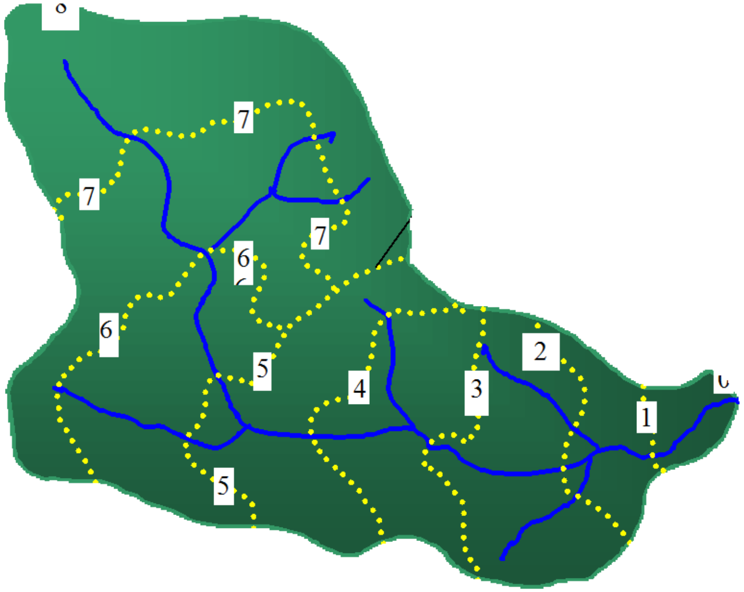


Time-Area



Hypothetical Example

- A 190 mi^2 watershed is divided into 8 isochrones of travel time.
- The linear reservoir routing coefficient, R, estimated as 5.5 hours.
- A time interval of 2.0 hours will be used for the computations.

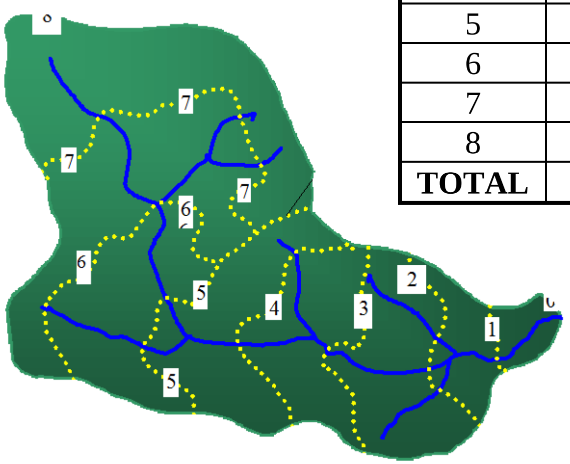


Rule of Thumb

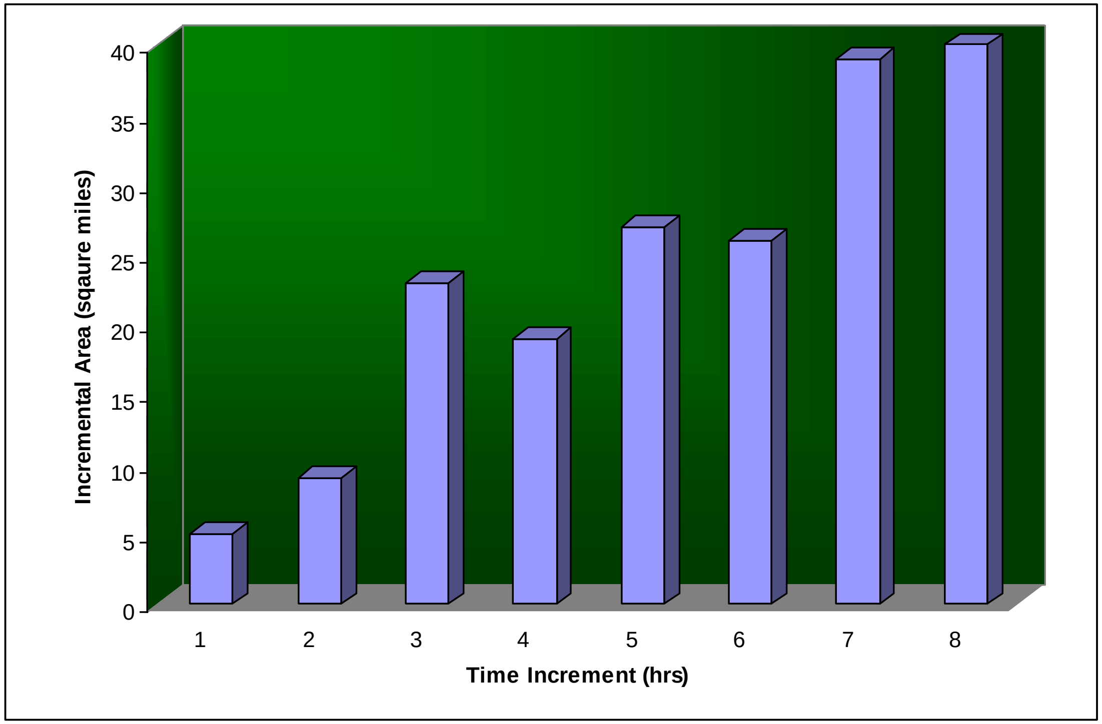
R - The linear reservoir routing coefficient can be estimated as approximately 0.75 times the time of concentration.

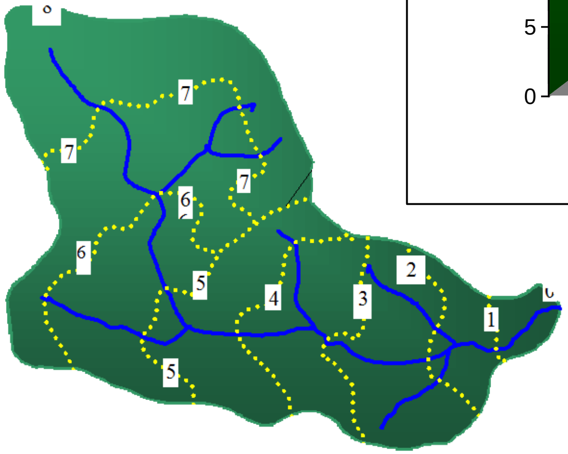
Basin Breakdown

Map Area #	Bounding Isochrones	Area (mi^2)	Cumulative Area (mi^2)	Cumulative Time (hrs)
1	0-1	5	5	1.0
2	1-2	9	14	2.0
3	2-3	23	37	3.0
4	3-4	19	58	4.0
5	4-5	27	85	5.0
6	5-6	26	111	6.0
7	6-7	39	150	7.0
8	7-8	40	190	8.0
TOTAL		**190**	**190**	**8.0**

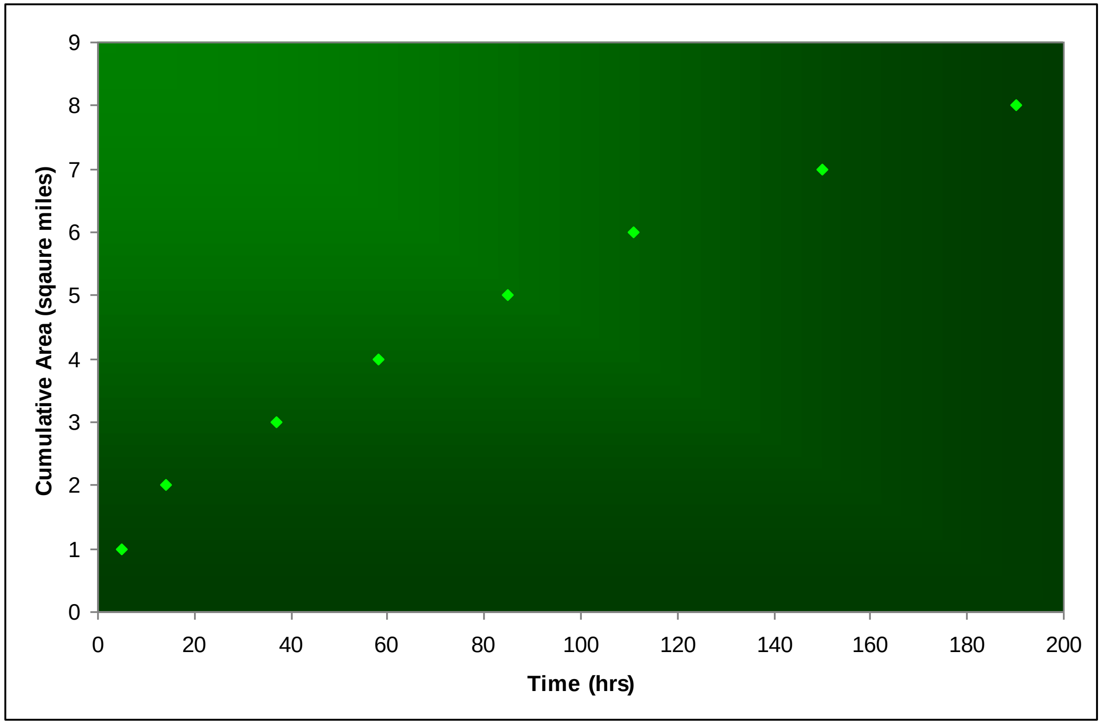


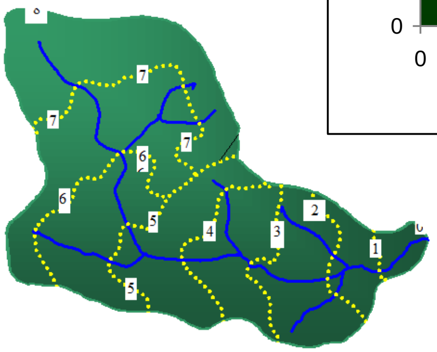
Incremental Area



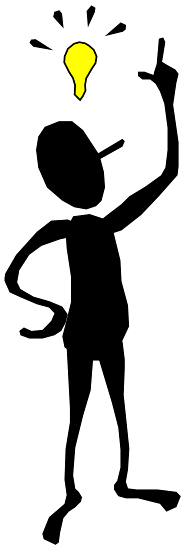


Cumulative Time-Area Curve





Trouble Getting a Time-Area Curve?



$$TA_i = 1.414 T_i^{1.5} \qquad \text{for } (0 \le \text{Ti} \le 0.5)$$

$$1 - TA_i = 1.414(1 - T_i)^{1.5} \qquad \text{for } (0.5 \le \text{Ti} \le 1.0)$$

Synthetic time-area curve - The U.S. Army Corps of Engineers (HEC 1990)

Instantaneous UHG

$$IUH_i = cI_i + (1-c)IUH_{(i-1)}$$

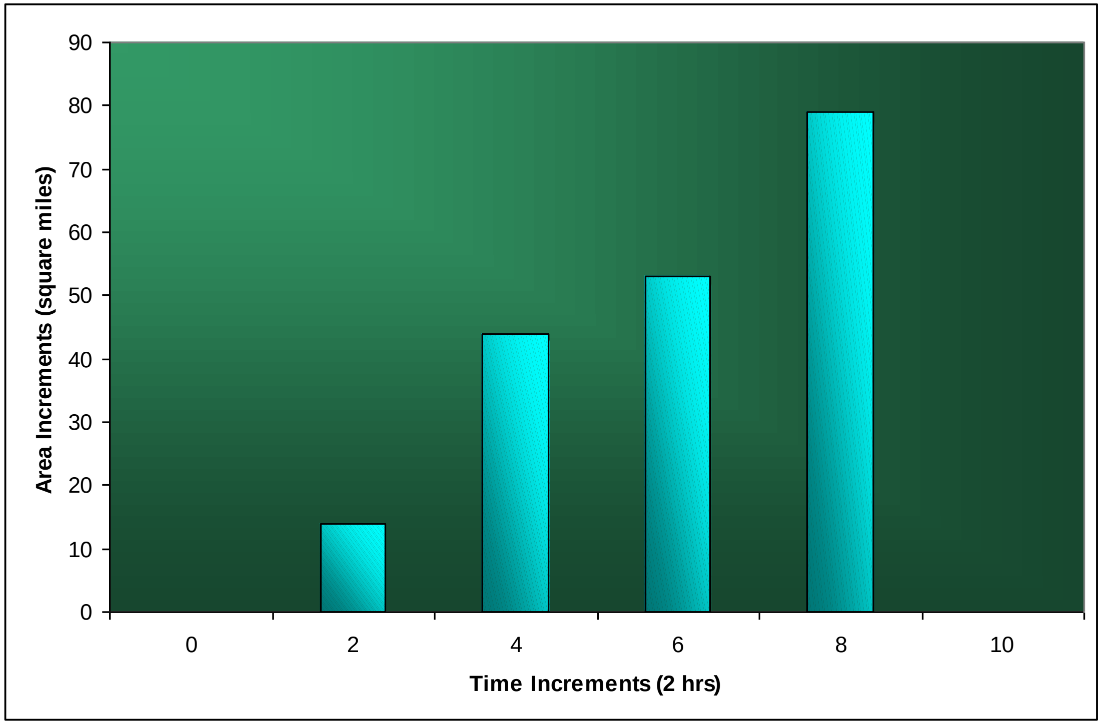
$$c = \frac{2\Delta t}{2R + \Delta t}$$

- ✓ **Δt = the time step used n the calculation of the translation unit hydrograph**
- ✓ **The final unit hydrograph may be found by averaging 2 instantaneous unit hydrographs that are a Δt time step apart.**

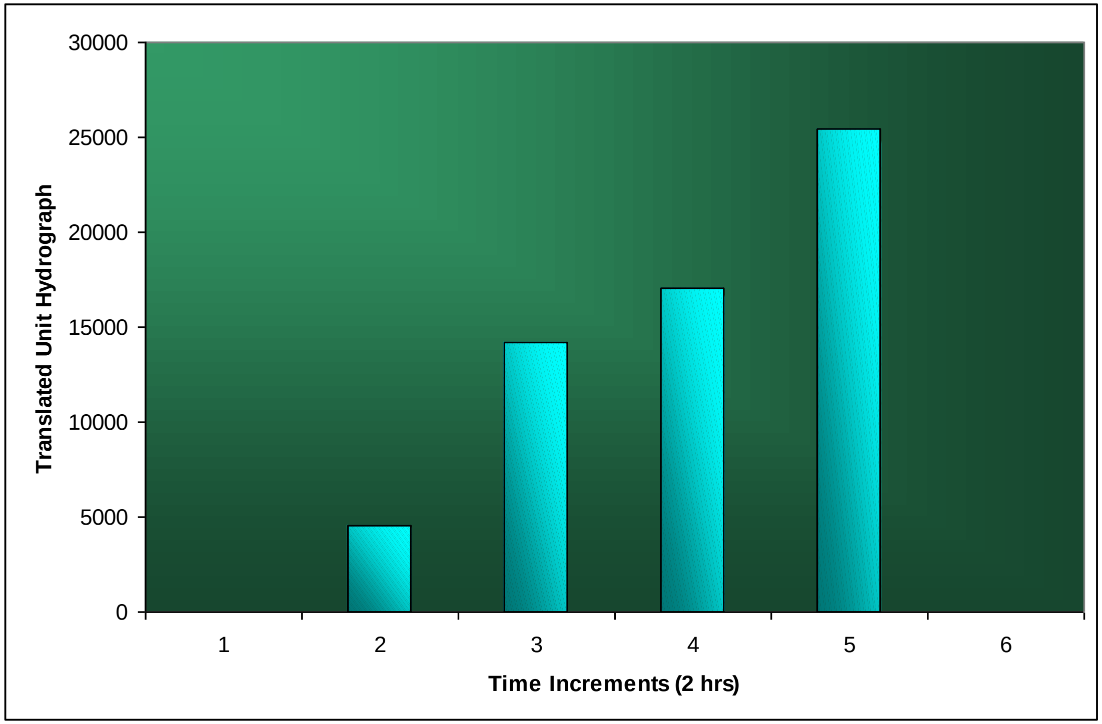
Computations

Time (hrs) (1)	Inc. Area (mi^2) (2)	Inc. Translated Flow (cfs) (3)	Inst. UHG (4)	IUHG Lagged 2 hours (5)	2-hr UHG (cfs) (6)
0	0	0	0		0
2	14	4,515	1391	0	700
4	44	14,190	5333	1,391	3,360
6	53	17,093	8955	5,333	7,150
8	79	25,478	14043	8,955	11,500
10	0	0	9717	14,043	11,880
12			6724	9,717	8,220
14			4653	6,724	5,690
16			3220	4,653	3,940
18			2228	3,220	2,720
20			1542	2,228	1,890
22			1067	1,542	1,300
24			738	1,067	900
26			510	738	630
28			352	510	430
30			242	352	300
32			168	242	200
34			116	168	140
36			81	116	100
38			55	81	70
40			39	55	50
42			26	39	30
44			19	26	20
46			13	19	20
48				13	

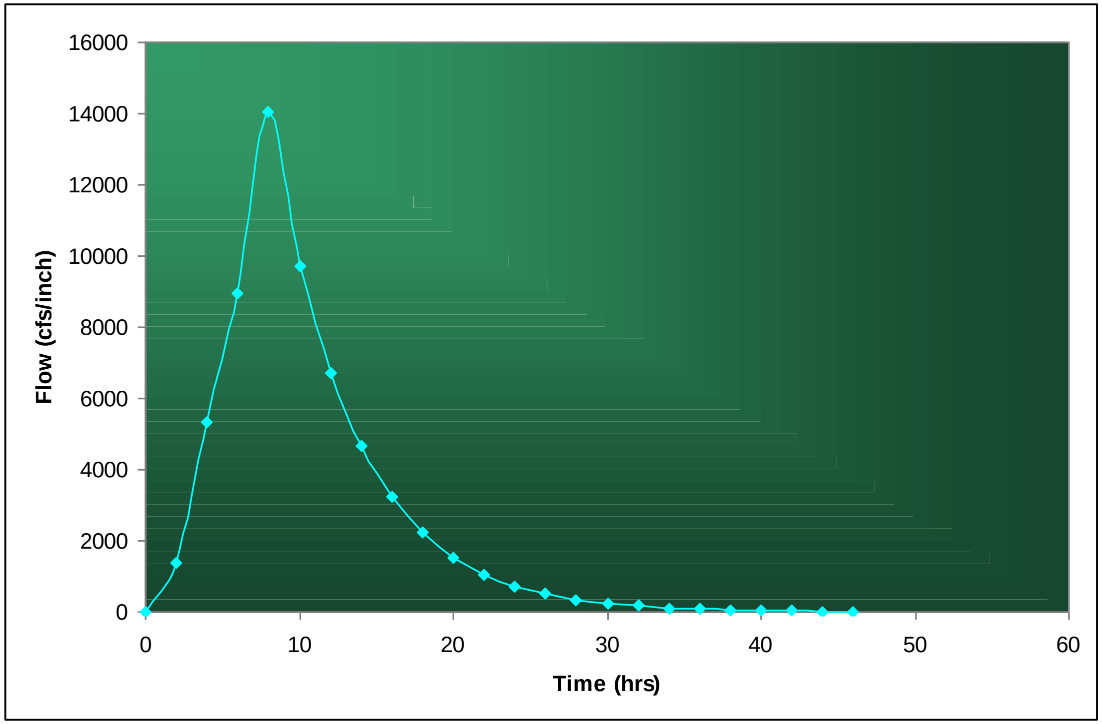
Incremental Areas



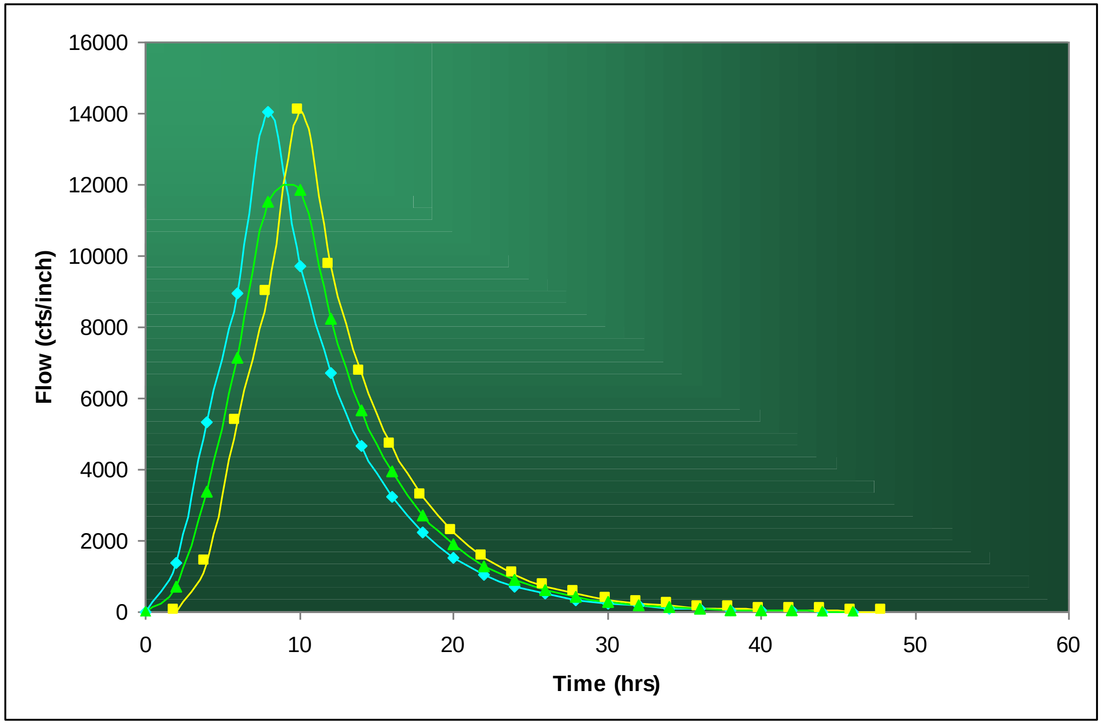
Incremental Flows



Instantaneous UHG



Lag & Average

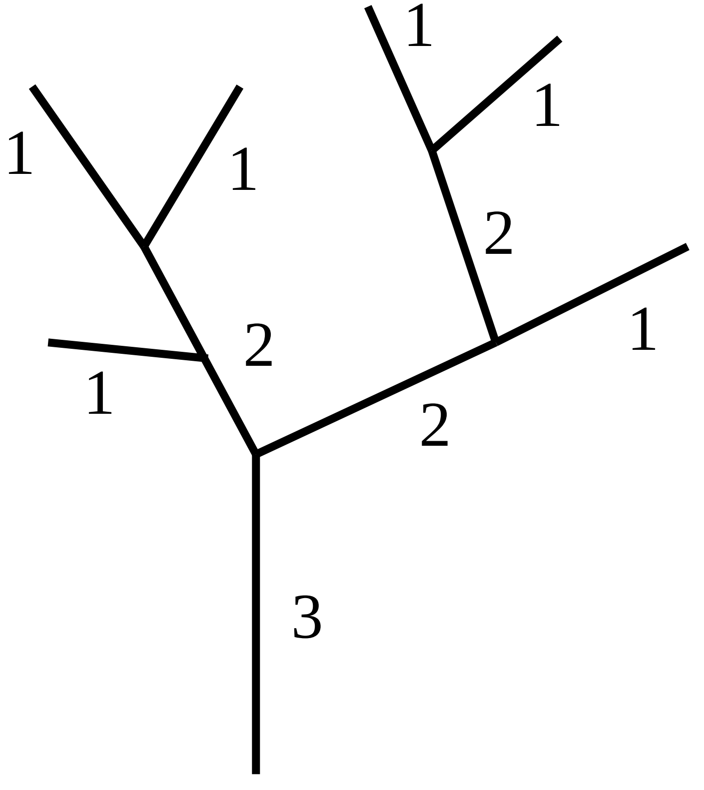


16000
14000
12000
10000
8000
6000
4000
2000
0
Flow (cfs/inch)
0
10
20
30
40
50
60
Time (hrs)

Geomorphologic

- Uses stream network topology and probability concepts
- Law of Stream Numbers $$\frac{N_{\omega-1}}{N_{\omega}} = R_B$$
  range: 3-5
- Law of Stream Lengths $$\frac{\overline{L_{\omega}}}{\overline{L_{\omega-1}}} = R_L$$
  range: 1.5-3.5
- Law of Stream Areas $$\frac{\overline{A_{\omega}}}{\overline{A_{\omega-1}}} = R_A$$
  range:3-6

Strahler Stream Ordering



Probability Concepts

- Water travels through basin, making transitions from lower to higher stream order
- Travel times and transition probabilities can be approximated using Strahler stream ordering scheme
- Obtain a probability density function analogous to an instantaneous unit hydrograph
- Can ignore surface/subsurface travel times to get a channel-based GIUH

GIUH Equations

- Channel-based, triangular instantaneous unit hydrograph:

$$q_p = \frac{1.31}{L_\Omega} R_L^{0.43} V \qquad t_p = \frac{0.44 L_\Omega}{V} \left(\frac{R_B}{R_A} \right)^{0.55} R_L^{-0.38}$$

- L_Ω in km, V in m/s, q_p in hr^{-1}, t_p in hrs